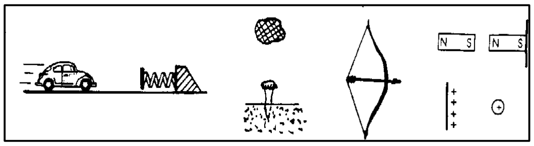
Work & Energy

An object is said to have energy if it has the ability to exert a force on another object over some distance.

For example:



1

WORK is the transfer of energy from one system or object to another.

When work is done on an object or system, its energy is either increased or decreased.

The transfer of energy is accomplished by the application of a force.

2

We'll look at two cases:

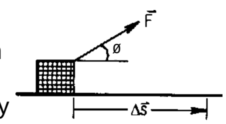
Case I) Work done by **CONSTANT** Forces

Case II) Work done by **VARYING** Forces including springs

3

Defining WORK Mathematically

The work done by a constant force F acting on an object resulting in a displacement s is given by



Work = (the component of F parallel to s)(s) because the perpendicular component does no work.

$W = F_{||}\ s = F\cos ø\ s$

where ł is the angle between the direction of **F** and the direction of **s**.

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In vector notation

$\boxed{W = F\cos ø\ s = \vec{F} \bullet \vec{s} \quad \text{(Dot product)}}$

Note that work and energy are SCALAR quantities

Work units => Force units • distance units
1 N • m = 1 Joule = 1 J

Do Vector Dot Products in Workbook Pg. 5

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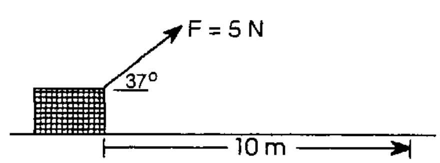
Work done by a CONSTANT FORCE

1. Find the work done in the following situation.
($\mu_k = 0$)



Ans. 40J

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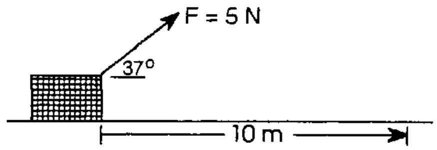
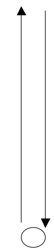
2. For the case shown below, let the mass of the block be 1.5 kg and μ_k = 0.2. What is the work done by gravity, normal force, the friction force, and net force as the block moves 10 meters?



Ans. $W_g = 0$ $W_f = -24$ J
$W_N = 0$ $W_F = +16$ J

3. A 0.25 kg ball is thrown vertically upward rising 10 meters before descending.

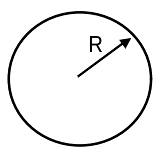
Find the work done on the ball by gravity:



I) as it ascends. Ans. -25J

II) as it descends. Ans. +25J

4. An object moves with uniform circular motion.



How much work is done on the object

i) during 1/4 revolution? Ans. i) 0

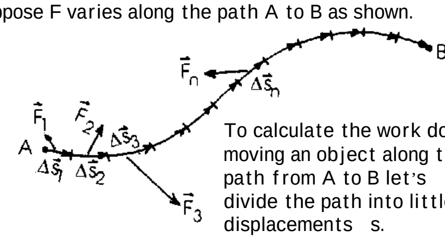
ii) during 1 revolution? Ans. ii) 0

Try Work - Thought problem #1 in workbook pg. 6.

Work done by a VARYING FORCE

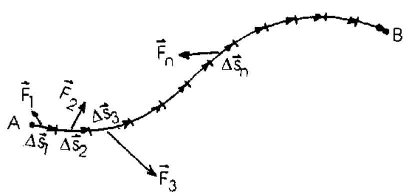
What if F is **not** constant?
What is the work done by F in this case?

Suppose F varies along the path A to B as shown.



To calculate the work done moving an object along the path from A to B let's divide the path into little displacements s.

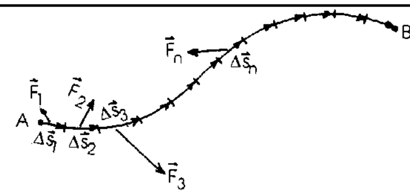
Let the s's be small enough that we can assume that F is constant over each small s.



The increment of work done in each segment is:

$W_1 = \mathbf{F}_1 \bullet \ \mathbf{s}_1$, $W_2 = \mathbf{F}_2 \bullet \ \mathbf{s}_2$,

$W_3 = \mathbf{F}_3 \bullet \ \mathbf{s}_3$, ... $W_n = \mathbf{F}_n \bullet \ \mathbf{s}_n$,



The total work by the force acting over the whole path is:

$$W_{total} = \sum \ \ W_n = \sum \mathbf{F}_n \ \ \mathbf{s}_n$$

If we make all the s's super teensy weensy then:

$$W\ (A\text{-->}B) = \int_A^B \mathbf{F} \cdot \mathbf{ds}$$ <--- The WORK Integral

Since $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y + \mathbf{F}_z$ and $d\mathbf{s} = d\mathbf{x} + d\mathbf{y} + d\mathbf{z}$,

$$W = \mathbf{F}_x \cdot d\mathbf{x} + \mathbf{F}_y \cdot d\mathbf{y} + \mathbf{F}_z \cdot d\mathbf{z}$$

However, at this level we'll only be calculating work done by forces that are just functions of **ONE** dimension.

That is, for this course:

$$W\ (A\text{-->}B) = \int_A^B \mathbf{F} \cdot d\mathbf{s}$$

where **F** is a constant or is a function of s and where s = x or y or z.

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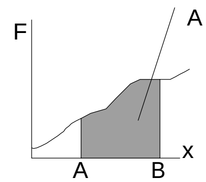
Recall that an integral also means the area under a curve.

Therefore, $$W\ (A\text{-->}B) = \int_A^B \mathbf{F} \cdot d\mathbf{s}$$

is the area under the Force vs. Displacement graph.



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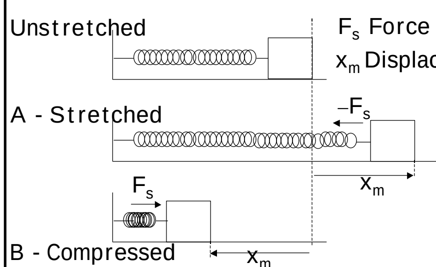
Work by a Spring

Springs are examples of systems with forces that vary with distance.



F_s Force of Spring = 0
x_m Displacement = 0

F is negative
x_m is positive

F is positive
x_m is negative

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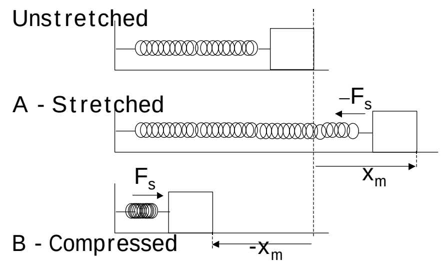

Hooke's Law for Spring Forces $\mathbf{F}_s = -k\mathbf{x}$
where k is spring constant (large k, stiff spring) and the negative is because $\mathbf{F}_s$ is always directed opposite of spring displacement.

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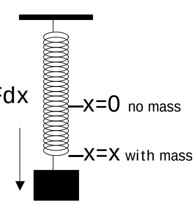
Example - Given a spring that obeys Hooke's law, $\mathbf{F} = -k\mathbf{x}$, where F is the magnitude of the force needed to stretch (or compress) the spring having a spring constant k by an amount x. Find the work done stretching the spring an amount x.

Solution by integration

$$W = \int_0^x F \cdot dx = \int_0^x F dx \cos 0^o = \int_0^x F dx$$

Since $\mathbf{F} = -k\mathbf{x}$,

$$W_s = \int_0^x -kx\, dx = -\frac{1}{2} kx^2$$



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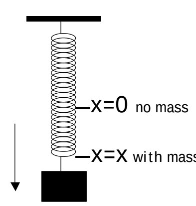
In summary

$$W = \int_0^x -kx\, dx = -\frac{1}{2} kx^2$$



Work by spring is negative
or energy is being removed
which means that Kinetic Energy decreases
or velocity is decreasing.
The spring slows down the mass.

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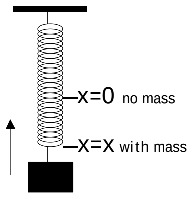
Now the spring is fully stretched and begins to compress BACK to x = 0.



Solution by integration

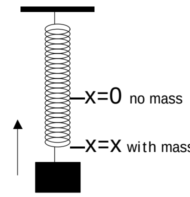
$$W = \int_x^0 \mathbf{F} \cdot d\mathbf{x} = \int_x^0 F dx \cos 0^o$$

$$= \int_x^0 F dx = \int_x^0 -kx\, dx = -\frac{1}{2} kx^2 \Big|_x^0$$

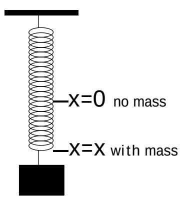
$$W = -\frac{1}{2} k(0^2 - x^2) = +\frac{1}{2} kx^2$$

In summary

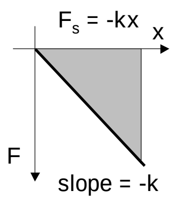
$$W = \int_x^0 \mathbf{F} \cdot d\mathbf{x} = +\frac{1}{2} kx^2$$



Work by spring is positive
or energy is being added
which means that Kinetic Energy increases
or velocity is increasing.
The spring accelerates the mass.



SAME Spring Example as previous



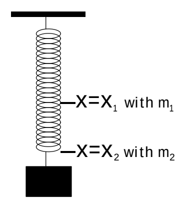
Solution by area under graph

$$W = \text{area} = \frac{1}{2} F (x - 0)$$

Since F = -kx for an extension x,

$$W_s = -\frac{1}{2} kx(x - 0) = -\frac{1}{2} kx^2$$

Same as before



One mass m_1 extends the spring to x_1 then adding more mass to a final m_2 extends the spring to x_2.

In other words, an additional force stretches the spring.

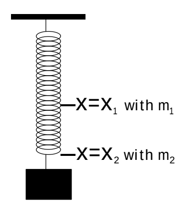
How much work is done by spring in stretching the spring from $x = x_1$ to $x = x_2$?

General equation for WORK

$$W = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} (-kx) dx = -\frac{1}{2} kx^2 \Big|_{x_i}^{x_f}$$

$$W_s = -\frac{1}{2} k(x_f^2 - x_i^2)$$

$$W_s = +\frac{1}{2} k(x_i^2 - x_f^2)$$



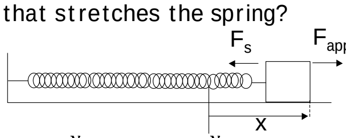
One mass m_1 extends the spring to x_1 then adding more mass to a final m_2 extends the spring to x_2.

Work by spring

$$W_s = +\frac{1}{2} k(x_i^2 - x_f^2)$$

$$W_s = -\frac{1}{2} kx_f^2 \quad \text{if } x_i = 0$$

What is the work done by APPLIED FORCE that stretches the spring?



If a = 0, then $\mathbf{F}_{app} = -\mathbf{F}_s$

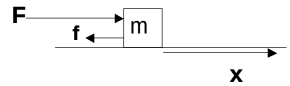
$$W = \int_0^x \mathbf{F}_{app} \cdot d\mathbf{x} = \int_0^x -F_s\, dx$$

$$= \int_0^x -(-kx)\, dx \quad = +\frac{1}{2} kx^2$$

$\boxed{\text{Note that } W_{Fapp} = -W_{spring}}$

WORK-ENERGY THEOREM

In the situation shown on the right, m will accelerate if



$\mathbf{F} = \mathbf{F}_{net}$ 0

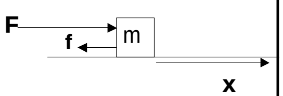
The net work done on the object is:

$W = W_F + W_f = \mathbf{F}\cdot\ \mathbf{x} + \mathbf{f}\cdot\ \mathbf{x} = (\mathbf{F} + \mathbf{f})\cdot\ \mathbf{x} = \mathbf{F}_{net}\cdot\ \mathbf{x}$

$\boxed{\text{Or simply } W_{net} = \mathbf{F}_{net}\cdot\ \mathbf{x}}$

Note that 1. When $\mathbf{F}_{net} = 0$, then $W_{net} = 0$ and a = 0.
2. When $\mathbf{F}_{net} \neq 0$, then $W_{net} \neq 0$ and $a \neq 0$.

Suppose $\mathbf{F}_{net}$ = constant 0, then



$\mathbf{F}_{net} = m\mathbf{a}$

$\mathbf{F}_{net}\cdot\ \mathbf{x} = m\mathbf{a}\cdot\ \mathbf{x}$

$F_{net} \cancel{\cos\phi}\ \ x = ma\cancel{\cos\phi}\ \ x$

From kinematics, $a\ \ x = \frac{1}{2}(v^2 - v_o^2)$

$W_{net} = \frac{1}{2} m (v^2 - v_o^2)$

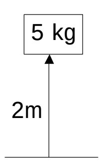
$\boxed{W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = \ (\frac{1}{2}mv^2) = \ \ KE}$

Definition: KINETIC ENERGY

$\boxed{KE = \frac{1}{2}mv^2}$ Energy of Motion

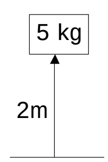
Units of KE => ML^2/T^2 => $kg\ m^2/s^2$
$= (kg\ m/s^2)\ m$
$= N\ m$
$= J$

WORK-ENERGY THEOREM
The NET work done on an object equals the resulting CHANGE in its KINETIC ENERGY
$W_{net} = \ \ KE$



Example:
Suppose a block is lifted 2m vertically at constant speed.

1. Determine the work done by the lifter. 100 J
2. Determine the work done by gravity. -100 J
3. Determine the net work done. 0 J
4. Determine KE. 0 J



Suppose a block is lifted 2m vertically at constant speed.

5. Use the WORK-ENERGY THEOREM (not kinematics) to find the speed of the block just before hitting the ground if it is dropped from a height of 2 m.

2 10 m/s

POWER

Power is defined as the time rate of doing work.

OR

Power is the rate at which energy is being transferred to or away from and object or system.

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Example: If 20 J of work is done on an object in 15 seconds, then the work is being done at a rate of

20 J / 15 s = 1.3 J/s

Thus the rate at which energy is being transferred to the object, or work being done per unit time, or POWER supplied to the object is 1.3 J/s.

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Definition: 1 Joule/sec = 1 Watt [W]
1000 W = 1 kW
746 W = 0.75 kW = 1 hp

Generally $\boxed{\text{Average Power } P_{ave} = \text{Work/time}}$

Since W = $\mathbf{F} \cdot d\mathbf{s}$, then $dW = \mathbf{F} \cdot d\mathbf{s}$

So $P_{inst} = dW/dt = \mathbf{F} \cdot d\mathbf{s} \,/\, dt = \mathbf{F} \cdot \mathbf{v}$

$\boxed{\text{Instantaneous Power } P_{inst} = dW/dt = \mathbf{F} \cdot \mathbf{v}}$

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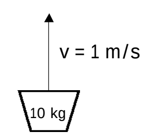
Example: Suppose you are pulling a 10 kg bucket of water out of a well at a constant speed of 1 m/s. Find the rate at which you are doing work or the power you are supplying to lift the bucket.

$P_{inst} = \mathbf{F} \cdot \mathbf{v}$

= (mg, up) • (v, up)

= (98 N, up)(1 m/s, up)cos 0

= 98 J/s = 98 W = 0.098 kW



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Summary

W = Fcosø s = **F** • **s** $\quad W(A \to B) = \int_A^B \mathbf{F} \cdot d\mathbf{s}$

$W_{net} = \mathbf{F}_{net} \cdot \mathbf{x}$

$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = (\frac{1}{2}mv^2) = KE$

$KE = \frac{1}{2}mv^2$

P_{ave} = Work/time $\quad P_{inst} = dW/dt = \mathbf{F} \cdot \mathbf{v}$

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