Name:____Per:____________

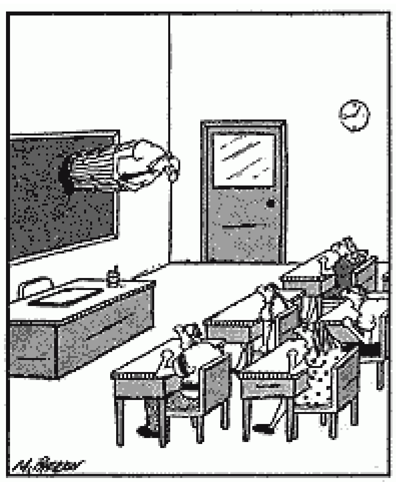
AP Physics C

Semester 1 - Mechanics

"Study and, in general the pursuit of truth and beauty is a sphere of activity in which we are permitted to remain children all of our lives."

Albert Einstein

Unit 1 Kinematics Workbook



"Good morning, and welcome to The Wonders of Physics."

Unit 1 - Kinematics

Supplements to Text Readings from
Fundamentals of Physics by Halliday, Resnick & Walker
<u>Chapter 2 & 4</u>

Unit 1 – Kinematics Objectives and Assignments

TEXT: *Fundamentals of Physics* by Halliday, Resnick, & Walker, **Chapter 2 & 4**

I) Motion in One Dimension

a. Students should understand the general relationship between position, velocity and acceleration for motion of a particle along a straight line, so that:

(1) Given a graph of one of the kinematics quantities, position, velocity, or acceleration, as a function of time, they can recognize in what time intervals the other two are positive, negative, or zero, and can identify or sketch a graph of each as a function of time.

(2) Given the expression for one of the kinematics quantities, position, velocity, or acceleration, as a function of time, they can determine the other two as a function of time, and find when these quantities are zero or achieve their maximum or minimum values.

b. Students should understand the special case of motion with constant acceleration so that they can:

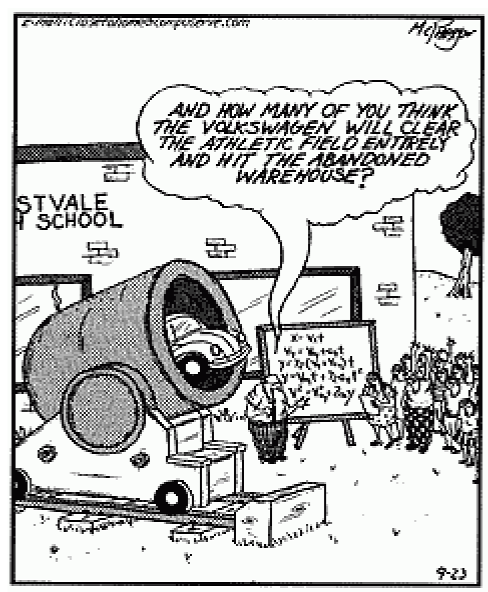
(1) Write down the expressions for velocity and position as functions of time, and identify or sketch graphs of these quantities.

(2) Use the equations

$$\mathbf{v} = \mathbf{v}_o + \mathbf{a}t \qquad \mathbf{d} = \mathbf{d}_o + \mathbf{v}_o t + \frac{1}{2}\mathbf{a}t^2 \qquad \mathbf{v}^2 = \mathbf{v}_o^{\,2} + 2\mathbf{a}(\mathbf{d} - \mathbf{d}_o)$$

to solve problems involving one-dimensional motion with constant acceleration.

c. Students should know how to deal with situations in which acceleration is a specified function of velocity and time so they can write an appropriate differential equation dv/dt = f(v)g(t) and solve it for v(t) incorporating correctly a given initial value of v.



Thanks to the innovative labs of teacher Herb Krenley, physics quickly became Westvale High's most popular course.

II. Motion in Two Dimensions

a. Students should be able to deal with displacement and velocity vectors so they can:

(1) Relate velocity, displacement, and time for motion with constant velocity.

(2) Calculate the component of a vector along a specified axis, or resolve a vector into components along two specified mutually perpendicular axes.

(3) Add vectors in order to find the net displacement of a particle that undergoes successive straight-line displacements.

(4) Subtract displacement vectors in order to find the location of one particle relative to another, or calculate the average velocity of one particle relative to another.

(5) Add or subtract velocity vectors in order to calculate the velocity change or average acceleration of a particle, or the velocity of one particle relative to another.

b. Students should understand the general motion of a particle in two dimensions so that given functions x(t) and y(t) which describes this motion, they can determine components, magnitude, and direction of the particle's velocity and acceleration as functions of time.

c. Students should understand the motion of projectiles in a uniform gravitational field so they can:

(1) Write down expressions for the horizontal and vertical components of velocity and position as functions of time, and sketch or identify graphs of these components.

(2) Use these expressions in analyzing the motion of a projectile that is projected above level ground with a specified initial velocity.

d. Students should understand the uniform circular motion of a particle so they can:

(1) Relate the radius of the circle and the speed or the rate of revolution of the particle to the magnitude of the centripetal acceleration.

(2) Describe the direction of the particle's velocity and acceleration at any instant during the motion.

(3) Determine the components of the velocity and acceleration vectors at any instant, and sketch or identify graphs of these quantities.

Mechanics Unit 1 Homework *Note additional problems will be assigned
Chapter 2 #4-9, 12, 13, 18, 19, 23-25, 27, 31, 34-42, 45-49, 52, 54, 59, 62
Chapter 4 #4, 8 ,12, 14-16, 23, 28, 33, 37, 39, 51, 64, 67

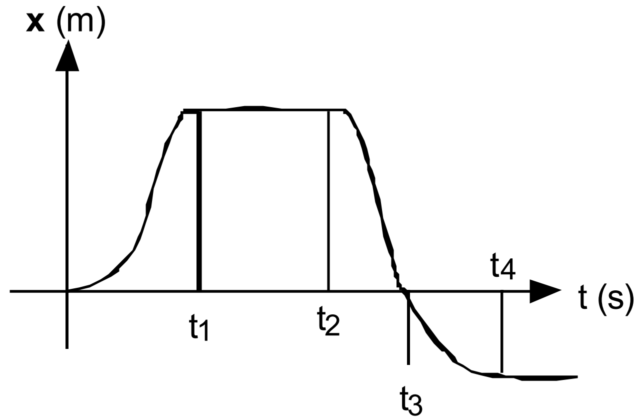
Graphs & Instantaneous Velocity

1. Given the position vs. time graph at the right, describe the motion. That is, when is the object moving with constant velocity?

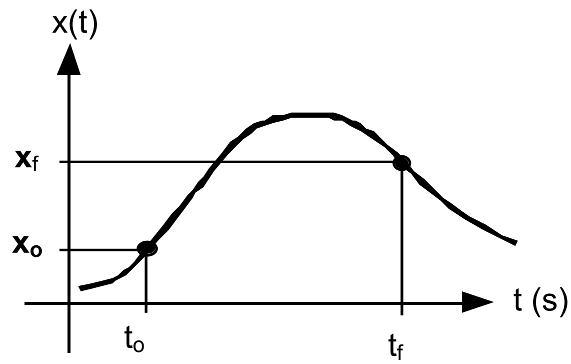
When is it accelerating?

…decelerating?

…at rest?



2. In terms of the quantities $\mathbf{x}_o$, $\mathbf{x}_f$, t_o, and t_f , what is the average velocity of the object having the position vs. time graph shown at the right?



3. What interpretation <u>in terms of the graph</u> can be placed on your answer above?

4. In # 3, you gave the definition of average velocity between two times t_o and t_f. What if you were asked “What is the velocity at a particular instant, say at t = 2s?”. Here you are being asked for the **INSTANTANEOUS VELOCITY**, $\mathbf{v}_{inst}$, which is defined as follows:

$$\boxed{v_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{x}}{\Delta t} = \frac{d\mathbf{x}}{dt}}$$

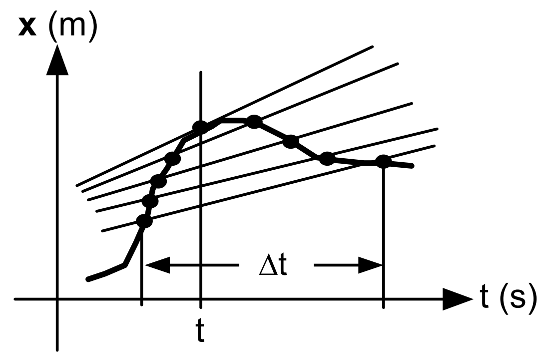
In everyday language, this says that the instantaneous velocity of an object is the value of the average velocity (since $\mathbf{v}_{ave} = \Delta\mathbf{x}/\Delta t$) as we make the time interval Δt super teensy-weensy.

Notice from the graph at the right that this has a simple graphical interpretation:

As we make Δt teensier and weensier then $\Delta\mathbf{x}/\Delta t$ merely becomes ________________

___________________________________.

This is exactly the interpretation of the derivative in Calculus.



5. Write your graphical interpretation of a derivative of dx/dt in your own words.

6. From Calculus, how do find the derivative of a function such as $x(t) = ax^n$?

x’ = dx/dt = _________________

7. Suppose that the velocity of an object changes with time according to the equation

$x = 0.25\, t^2 + 1$, where x is in m and t is in seconds.

a) Derive the expression for the instantaneous velocity[1] of the object at any time t and use it to calculate the velocity at t = 0 and t = 6s.

$\mathbf{v}_{inst} = d\mathbf{x}/dt$ = _____________

so **v** (t=0) = _______ **v** (t=6) = ____________

[1] 0.5t

7. b) Plot the position of the object versus elapsed time between t = 0 and t = 8s.

x	t

c) Use your graph above and find the **velocity** of the object at t = 0 and t = 6 s by **approximating** the slope at these times.

v (t=0) ≈ ________ **v** (t=6) ≈ _____________

d) How do the values of the velocity found from the slope compare to the values of the velocity found using the equation in part a)?

Instantaneous Velocity vs. Average Velocity Lab

PURPOSE

The purpose of this lab is to investigate the relationship between instantaneous velocity and average velocity, and see how a series of average speeds can be used to deduce an instantaneous velocity

THEORY

An average velocity can be a useful value. It's the ratio of the overall distance an object travels and the amount of time that the object travels. If you know you will average 50 miles per hour on a 200 mile trip, it's easy to determine how long the trip will take. On the other hand, the highway patrol officer following you doesn't care about your average velocity over 200 miles. The patrol officer wants to know how fast you're driving at the instant the radar strikes your car, so he or she can determine whether or not to give you a ticket. The officer wants to know your instantaneous velocity. If you measure average velocity of a moving object over smaller and smaller intervals of distance, the value of the average velocity approaches the value of the object's instantaneous velocity.

MATERIALS & PROCEDURES

Given that you understand the theory above together with the discussions in class and with instructions of how to use computer interfaces with photogates and picket fences, your groups should be able to decide what kind of data you need to take and how to analyze it. There are prepared instructions that come with this lab and are available if your group has no idea what to do. However, you will learn a lot more if you do this lab without the aid of instructions.

DATA

Your procedures will determine what kind of raw data you will report.

ANALYSIS

1. Which of the average speeds that you measured gave you the closest approximation to the instantaneous velocity? Why?

2. What is the relationship between the "Y-Intercept" on your graph and the instantaneous velocity of the cart?

3. What factors influence your results? Discuss how each factor influences the result.

4. Are there ways to measure the instantaneous velocity directly, or is instantaneous velocity always a value that must be derived from average velocity measurements?

5. Other observations and analysis.

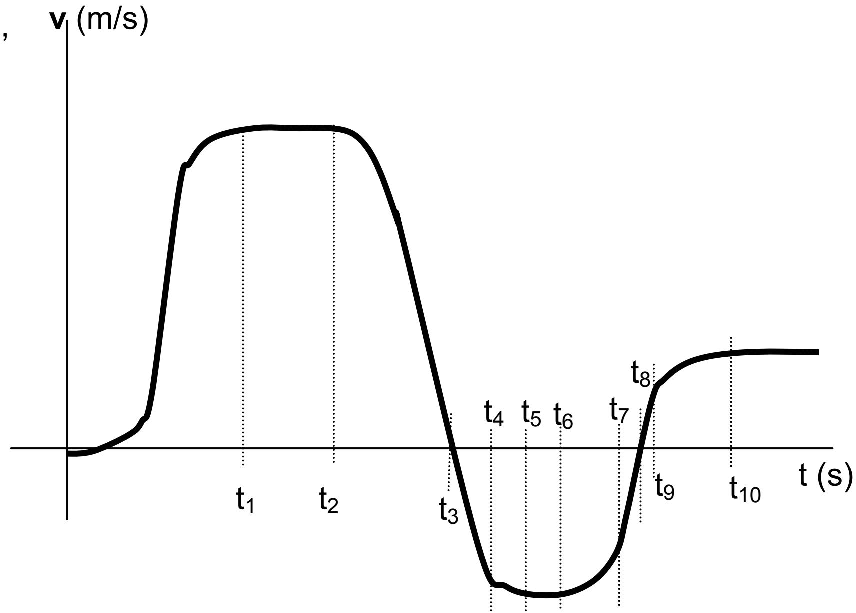
CONCLUSIONS

Include but don't limit your discussion to:

- What is important to take away from this lab?
- Summarize your purpose, results, analysis and findings.

Graphs & Instantaneous Acceleration

1. Given the velocity vs. time graph at the right, describe the motion. That is,



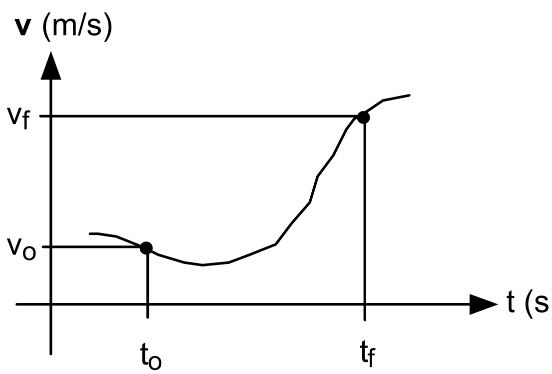
   a) When is the object moving with constant velocity?

   b) When is it accelerating?

   c) When is it decelerating?

   d) When is it at rest?

2. In terms of the quantities $\mathbf{v}_o$, $\mathbf{v}_f$, t_o, and t_f , what is the average acceleration of the object having the velocity vs. time graph shown at the right?



3. What interpretation in terms of the graph can be placed on your answer above?

In # 3, you gave the definition of average acceleration between two times t_o and t_f. What if you were asked "What is the acceleration at a particular instant, say at t = 2s?". Here you are being asked for the **INSTANTANEOUS ACCELERATION**, $\mathbf{a}_{inst}$, which is defined as follows:

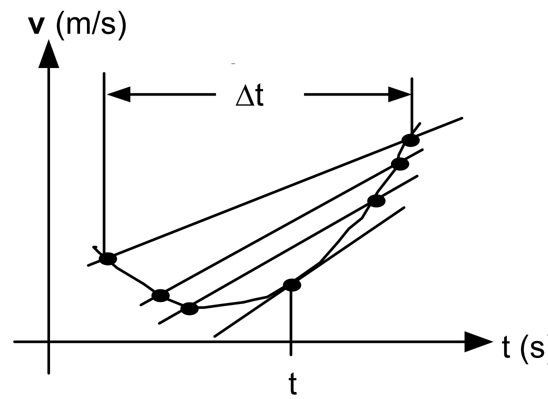
$$\boxed{\mathbf{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}}$$

where the direction of $\mathbf{a}_{inst}$. is the same direction as the direction of the **CHANGE** in velocity, $\Delta\mathbf{v}$.

Again, in everyday language, this says that the instantaneous acceleration of an object is the value of the average acceleration (since $\mathbf{a}_{ave} = \Delta\mathbf{v}/\Delta t$) as we make the time interval Δt super teensy-weensy.

4. The graphical interpretation of this definition is similar to the previous interpretation of $\mathbf{v}_{inst}$. Notice from the graph at the right that this has a simple graphical interpretation:

> As we make Δt teensier and weensier then $\Delta\mathbf{v}/\Delta t$ merely becomes ________________
> ____________________________________.
>
> This is again the interpretation of the derivative dv/dt in Calculus.



5. Write your graphical interpretation of a derivative of dv/dt in your own words.

6. Suppose that the **velocity** of an object changes with time according to the equation

$v = 0.5\ (1 + t^3)$, where v is in m/s and t is in seconds.

a) Derive the expression for the instantaneous acceleration of the object at any time t and use it to calculate the acceleration at t = 0 and t = 3s.

$\mathbf{a}_{inst} = d\mathbf{v}/dt =$ ______________

a (t=0) = _______ **a** (t=3) = ____________

6. b) Plot the velocity of the object versus elapsed time between t = 0 and t = 4s.

v	t

c) Use your graph above and find the **acceleration** of the object at t = 0 and t = 3 s by finding **approximating** the slope at these times.

a (t=0) = ________ **a** (t=3) = ______________

d) How do the values of the acceleration found from the slope compare to the values of the acceleration found using the equation in part a)?

Elementary Calculus – Differentiation

Find the 1st derivative of the following functions:

1. $y = 6$
2. $y = 8t^6$
3. $y = 1 / t^3$
4. $y = 2t - 5t^2$
5. $y = 4t^{-3}+2t^{1/2}$
6. $y = 3t^5(t^2+2)$
7. $y = (t^2-2)^2$
8. $y = (1+t)^{1/2}$
9. $y = (2+3t^2)^{1/3}$
10. $y = (3t+2t^2)^{-3}$
11. $y = (t^2-t^{-3})^2$
12. $y = t / (1-2t)$
13. $y = (2t^2-1) / t^2$
14. $y = (3t^2+t-1) / (6t^2-2)$
15. $y = 1 / (6t^2-2)$
16. $y = 2 \cos^2 t$
17. $y = 6 \sin(2t+3)$
18. $y = t+\cos^3 t$
19. $y = t^2 \cos(3t-1)$
20. $y = \sin t / t$
21. $y = 2 \ln t^2$
22. $y = t \ln t^3$
23. $y = -3\ln(t-1)$
24. $y = 2e^{-4t}$
25. $y = -4e^{2t}$
26. $y = 5e^{t+1}$

27. Find the 2nd derivative of the function given in problem #4.

28. Find the 2nd derivative of the function given in problem #17.

29. Find the 3rd derivative of the function given in problem #3.

30. Find the 3rd derivative of the function given in problem #5.

Maxima & Minima

How do you find the relative maxima and minima of a function because in physics, the physical interpretation those points are the most interesting and imply a significant change in the system?

MAIN CONCEPT: An extrema (minima or maxima) occurs for a function when the first derivative is zero. For example, for a function of position versus time, x(t), we know the first derivative, x'(t) of the function gives the velocity. The position is an extrema when the velocity is zero.

STEPS TO FIND EXTREMA

1. Set the 1st derivative of the function equal to zero.
2. Solve the 1st derivative for the unknown. The value(s) for the unknown is/(are) the critical point(s) at which the function is an extrema (x_c).
3. To test if the extrema is a maxima or minima, solve for the 2nd derivative and plug in the critical point(s) from step 2.
   -If the value for the 2nd derivative is **positive**, then the critical point is a **minima**.
   -If the value for the 2nd derivative is **negative**, then the critical point is a **maxima**.
   -If the value for the 2nd derivative is **zero**, the test is **inconclusive**.

Determine the extrema of the following functions:

1. $f(x) = -2x^3 + 15x^2 - 36x + 7$

Step 1: $f'(x) =$

Step 2:

Step 3: $f''(x_c) =$

(min. at x = 2, max. at x = 3)

2. $f(x) = 3x^5 - 20x^3$

Step 1: $f'(x) =$

Step 2:

Step 3: $f''(x_c) =$

(min. at x = 2, max. at x = -2)

Find the coordinates of all the maxima and minima of the following functions. State which are maxima and which are minima.

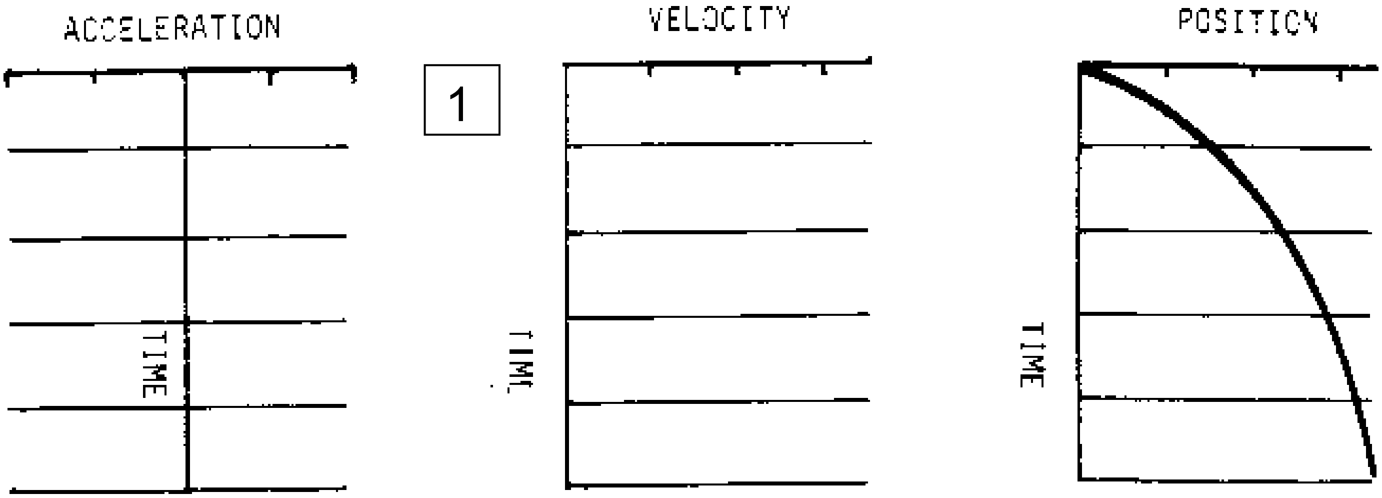
4)[2] $y = t^3-6t^2+9t$

5)[3] $y = 2t^2-t^4$

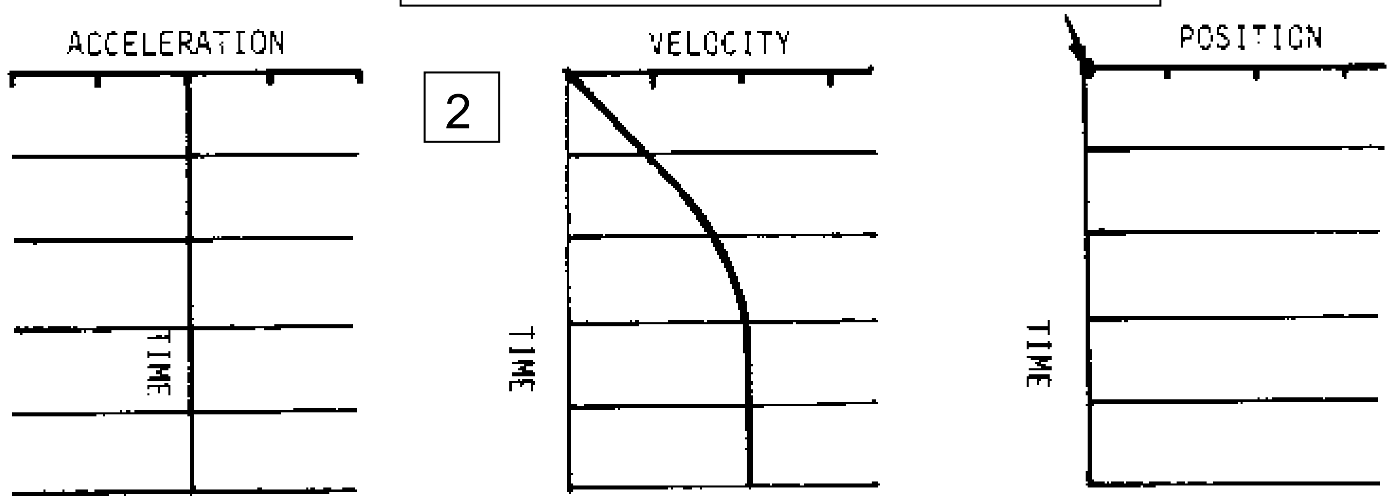
6) $y = t^3 + 10t^2 +25t - 25$

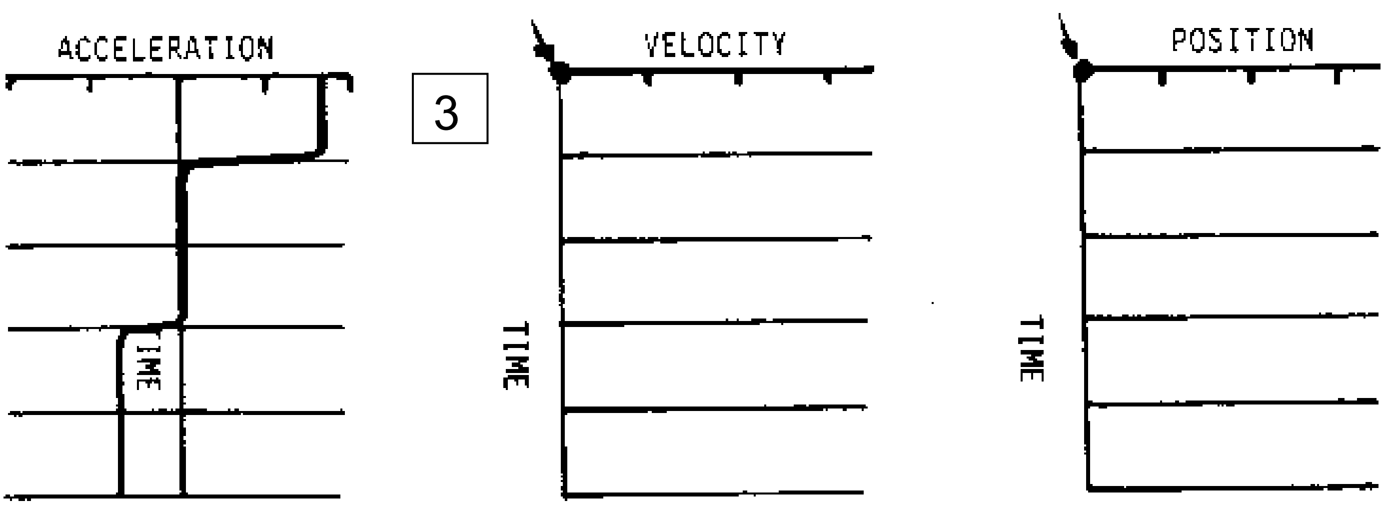
[2] max: (1,4), min: (3,0)
[3] max: (1,1)&(-1,1), min: (0,0)

Kinematics Graphs



Draw the other two graphs given one of the three.
Start at origin or where arrow indicates.





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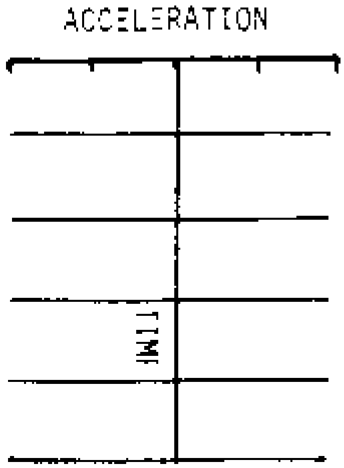
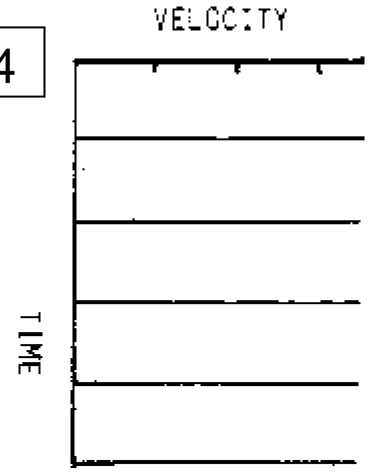
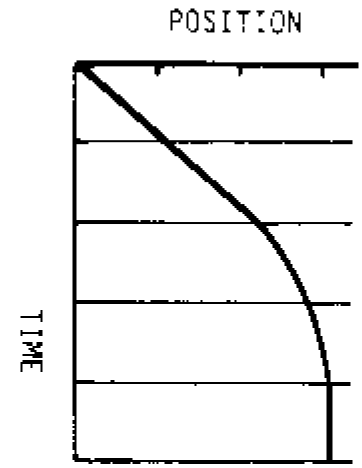
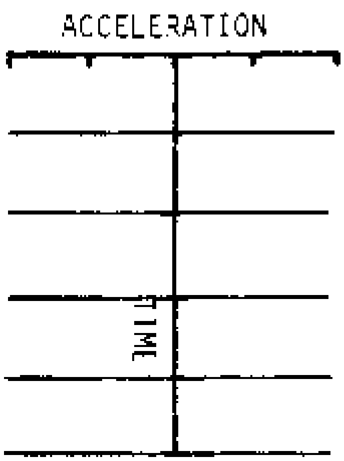
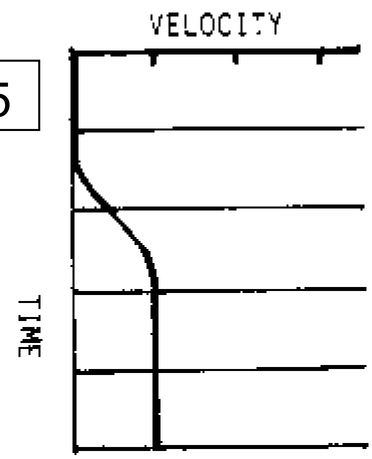

ACCELERATION
TIME

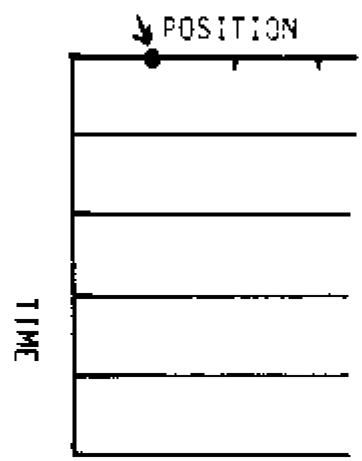

VELOCITY
TIME


POSITION
TIME

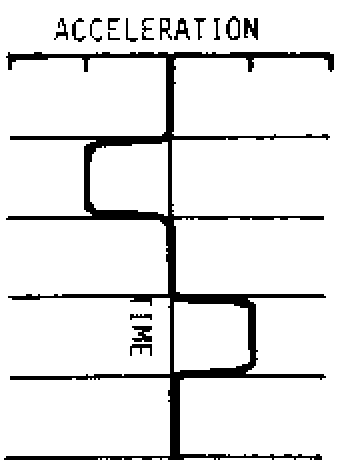
5

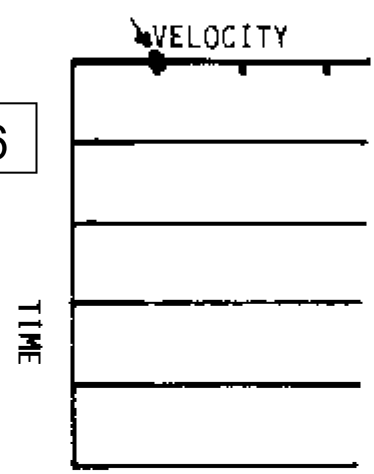

ACCELERATION
TIME

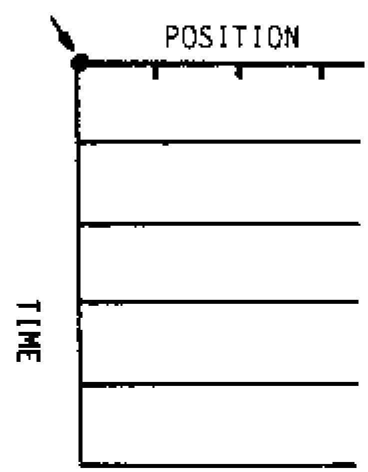

VELOCITY
TIME


POSITION
TIME

6


ACCELERATION
TIME


VELOCITY
TIME


POSITION
TIME

Kinematics Practice with Calculus - Differentiation

1. The position of an object moving along a straight line is given by $x = 3 - 2t^2 + 3t^3$ where x is in meters and t in seconds. **SHOW ALL WORK and/or EXPLAIN IN DETAIL!**

   a) Derive the expressions for the velocity and acceleration of the object as a function of time. ($v = -4t + 9t^2$, $a = -4 + 18t$)

   b) Find the position of the object at t = 0, t = 2s, t = 4s. (3m, 19m, 163m)

   c) Find the displacement or the object between t = 2s and t = 4s; between t = 0s and t = 4s. (144m, 160m)

   d) Find the average velocity between t = 2s and t = 4s; between t = 0s and t = 4s; between t = 1s and t = 3s. (72m/s, 40m/s, 31m/s)

   e) What is the instantaneous velocity at t = 2s? at t = 5s? (28m/s, 205m/s)

   f) At what time(s) is/are the instantaneous velocities zero? (0s, 0.44s)

g) When does the instantaneous velocity have a maximum or a minimum value? (0.22s)

h) Find the change in velocity between t = 2s and t = 5s. (See part e) (177m/s)

i) Find the average acceleration between t = 2s and t = 5s; between t = 1s and t = 3s. ($59 m/s^2$, $32 m/s^2$)

j) When is the instantaneous acceleration of the object zero? (0.22s)

k) Find the instantaneous acceleration of the object at t = 2s; t = 5s. ($32 m/s^2$, $86 m/s^2$)

2. The position of a body moving along a straight line is given by $x = 16t - 6t^2$ where x is in meters and t in seconds.

a) Find the position of the body at t = 1s. (10m)

b) At what times does the body pass the origin? (0s, 2.67s)

c) Calculate the average velocity of the body between t = 0 and 2 seconds. (4m/s)

d) Find the velocity of the object at any time t. (v = 16 - 12t)

e) What is velocity at t = 0? at t = 2s? (16m/s, -8m/s)

f) At what times and positions will the body be at rest? (1.33s, 10.6m)

g) Find the acceleration of the body at any time t. ($-12 m/s^2$)

h) When is the acceleration of the body zero? (never)

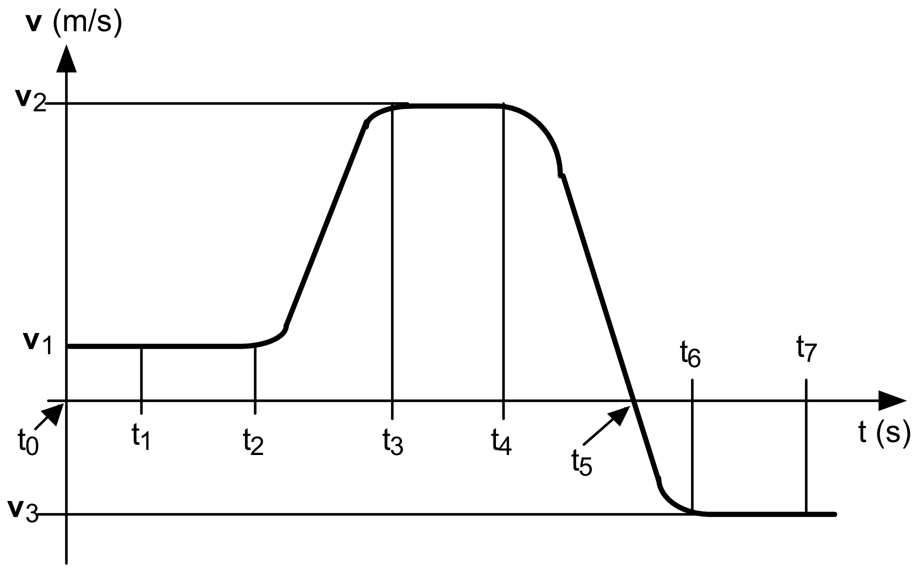
i) Using the data calculated from a-h, plot a vs t, v vs t, and x vs t.

j) During what time interval(s) is the body "speeding up" (i.e. accelerating)? ($t > 1.33$s)

k) During what time interval(s) is the body "slowing down" (i.e. decelerating)? ($0 < t < 1.33$s)

Areas UNDER Velocity Graphs

Suppose we wish to find the area between the graph and the time axis on the graph below between $t = t_o$ and $t = t_1$. The area = height x base = $(\mathbf{v}_1)(t_1 - t_o) = \mathbf{v}_1 \bullet \Delta t$



But from the definition of $\mathbf{v}_{ave}$: $\Delta\mathbf{x} = \mathbf{v} \bullet \Delta t$, if $\mathbf{v}$ is constant over the interval Δt. Therefore, we can see that the area between the $\mathbf{v}$ vs. t graph and the t axis gives the displacement of the body during the time interval t. Or in short:

Area ($\mathbf{v}$ vs. t) = $\Delta\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_o = \mathbf{v}_1 (t_1 - t_o)$
Area ($\mathbf{v}$ vs. t) = the displacement between $t = t_o$ and $t = t_1$.

Referring to the graph above, answer the following questions:

1. What does the area under the graph and the t axis between t_1 and t_2 represent?
____

2. What is the displacement of the object between t_2 and t_3?

$\Delta\mathbf{x}_{2\text{-}3}$ = ____

3. What is the displacement of the object between t_0 and t_4?

$\Delta\mathbf{x}_{0\text{-}4}$ = ____

4. Suppose the graph on the previous page is described by $v(t) = kt^2 + b$ between t_4 and t_5. How would you find the displacement between t_4 and t_5?

Since velocity is the derivative of the position with respect to time, through calculus, we can also find the displacement by finding a position function whose derivative fits the function for velocity. This process is called integration (or antidifferentiation).

DISPLACEMENT from a function of velocity is given by $\Delta x = (\mathbf{x}_2 - \mathbf{x}_1) = \int_{t_1}^{t_2} \mathbf{v}(t)dt$

The above is just a fancy way of calculating the AREA BETWEEN THE VELOCITY VERSUS TIME AXIS BETWEEN TIMES t_1 and t_2.

So to find the displacement between t_4 and t_5, you must integrate the above velocity function with limits of integration between t_4 and t_5.

$\Delta \mathbf{x}_{4\text{-}5} = \int \mathbf{v}(t)\, dt = \int_{\_\_}^{\_\_}$ ___________ dt = ____________________

$\Delta \mathbf{x}_{4\text{-}5} =$ ______________________________

5. a) From the graph on the previous page, what is the displacement between t_5 and t_6?

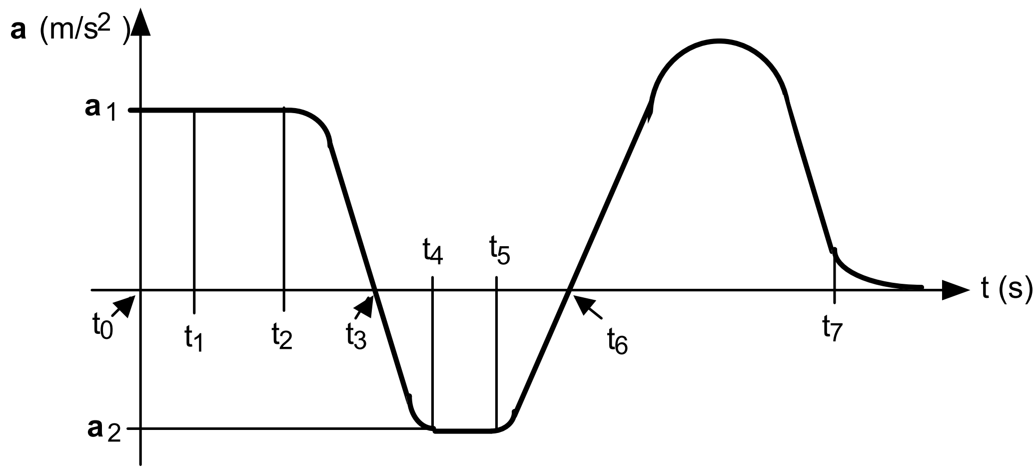
b) Your answer in part a) should be negative. What physical meaning does this negative sign mean for the object?

6. How would you find the total displacement of the object between t_0 and t_7?

Areas UNDER Acceleration Graphs

Given the acceleration versus time graph below, what is the area between the acceleration line and the time axis between $t = t_0$ and $t = t_1$?

Area from t_0 to t_1 = ________________________________



Your answer should have been

Area from t_0 to t_1 = $\mathbf{a}_1 (t_1 - t_0) = \mathbf{a}_1 \cdot \Delta t$

But from the definition of $\mathbf{a}_{ave}$: $\Delta\mathbf{v} = \mathbf{a} \cdot \Delta t$, if **a** is constant over the interval Δt. Therefore, we can see that the area between the **a** vs. t graph and the t axis gives the change in velocity of the body during the time interval t_0 to t_1. Or in short:

Area (**a** vs. t) = $\Delta\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_o = \mathbf{a}_1 (t_1 - t_o)$
Area (**a** vs. t) = the change in velocity between $t = t_o$ and $t = t_1$.

Look familiar?

Referring to the graph above, answer the following questions:

1. What does the area under the graph and the t axis between t_1 and t_2 represent?

____

2. What is the change in velocity of the object between t_2 and t_3?

$\Delta\mathbf{v}_{2\text{-}3}$ = ____

3. What is the change in velocity of the object between t_0 and t_6?

$\Delta \mathbf{v}_{0\text{-}6}$ = ____

4. Suppose the graph on the previous page is described by $a(t) = kt^2 + b$ between t_6 and t_7. How would you find the change in velocity between t_6 and t_7?

Since acceleration is the derivative of the velocity with respect to time, through calculus, we can also find the velocity by finding a velocity function whose derivative fits the function for acceleration. Again, we use process of integration.

<u>Change in Velocity</u> from a function of acceleration is given by $\Delta v = (\mathbf{v}_2 - \mathbf{v}_1) = \int_{t_1}^{t_2} \mathbf{a}(t)dt$

where the resulting sign indicates the direction of the **CHANGE** in the velocity vector. The above is just a fancy way of calculating the AREA BETWEEN THE <u>ACCELERATION</u> VERSUS TIME AXIS BETWEEN TIMES t_1 and t_2.

5. So to find the change in velocity between t_6 and t_7, you must integrate the above velocity function with limits of integration between t_6 and t_7.

$\Delta \mathbf{v}_{6\text{-}7} = \int \mathbf{a}(t)\,dt = \int_{\_\_}^{\_\_}$ ___________ dt = ____________________

$\Delta \mathbf{v}_{6\text{-}7}$ = ______________________________

Elementary Calculus - Integration

Practice finding the following indefinite integrals. **Use the formulas on Pg. A12 in your book.**

1. $\int 4t\ dt$

2. $\int \frac{dt}{t^3}$

3. $\int \left(2t^3 + 3t^{1/2}\right) dt$

4. $\int \frac{dt}{1-4t}$

5. $\int 2e^{-4t}\ dt$

6. $\int \sin^2 t(\cos t)\ dt$

7. $\int t \sin t^2\ dt$

8. $\int 8 \cos 3t\ dt$

9. $\int t\ (t^2 - 3)^{1/2}\ dt$

10. $\int 3t^{-3/2}\ dt$

Evaluate the following definite integrals.

1. $\int_{0}^{1} 2t \, dt$

2. $\int_{1}^{10} (4 + 2t) \, dt$

3. $\int_{0}^{\pi/2} \sin(t) \, dt$

4. $\int_{0}^{\pi} [t + \sin(2t)] \, dt$

5. $\int_{-1}^{1} (t + t^2 + t^3) \, dt$

6. $\int_{0}^{\infty} (-6e^{-3t}) \, dt$

Kinematics Practice with Calculus - Integration

1. The acceleration of a body in 1-D motion is given by $a = 4 - t^2$
where a is in m/s^2 and t is in seconds.

If at t = 3s, v = 2m/s, and x = 9 m,

a) derive the expression for the velocity of the body as a function of time. $(v = -1 + 4t - t^3/3)$

b) derive the expression for the position of the body as a function of time. $(x = 0.75 - t + 2t^2 - 0.083\ t^4)$

2. The acceleration of a body moving along a straight line is given by $a = 6t – 3$
where a is in m/s^2 and t is in seconds.
At t = 0, the position of the body is x_o and its velocity is v_o.

a) When is the acceleration of the body zero? (0.5s)

b) Derive the expression for the velocity of the body at any time in terms of v_o and t. $(v = v_o + 3t^2 - 3t)$

c) If v_o = 2m/s, find the velocity of the body at t = 2s. (8m/s)

d) When is the velocity of the body a minimum and what is its value? (0.5s, 1.25m/s)

e) Derive the expression for the position of the body at any time in terms of x_o and t.
($x = x_o + 2t + t^3 - 1.5\, t^2$)

f) Find the displacement of the body between t = 0 and t = 5s. (97.5 m)

g) If x_o = -1m, find the position of the body at t = 3s. (18.5m)

3. A body is moving along a straight line according to the relation $v = t^3 + 4t^2 + 2$

If $x = 4$ m when $t = 2$s,

a) find the position of the body at any time t. ($x = t^4/4 + 4t^3/3 + 2t - 14.67$)

b) find the position of the body at $t = 3$s. (47.6m)

c) find the acceleration of the body at any time t. ($a = 3t^2 + 8t$)

d) find the acceleration of the body at $t = 3$s. (51 m/s^2)

4. (This one's HARD!!!) For the body in rectilinear motion whose acceleration is given by

$$a = 32 - 4v$$

derive an expression for its velocity as a function of time and its position as a function of time. All quantities are in MKS units and the initial conditions are that $x = 0$ m and $v = 4$ m/s at $t = 0$.

($v = 8 - 4e^{-4t}$, $x = 8t + e^{-4t} - 1$)

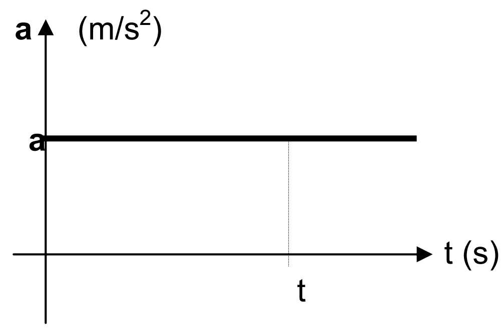
Deriving Kinematics Equations

We'll apply what we've learned graphically to an important physical case of an object moving at <u>constant acceleration</u>. Here you will derive the equations of motion below for an object falling vertically under the influence of gravity, an object moving horizontally under the influence of a constant unbalanced force, or any situation where the acceleration of the object is constant.

Equations of motion for <u>constant acceleration</u>

Eqn. I:	$\mathbf{v} = \mathbf{v}_o + \mathbf{a}t$	missing $\mathbf{x}$ or $\mathbf{x}_o$
Eqn. II:	$\mathbf{x} = \mathbf{x}_o + \frac{1}{2}(\mathbf{v} + \mathbf{v}_o)\, t$	missing $\mathbf{a}$
Eqn. III:	$\mathbf{x} = \mathbf{x}_o + \mathbf{v}_o t + \frac{1}{2}\mathbf{a}t^2$	missing $\mathbf{v}$
Eqn. IV:	$\mathbf{v}^2 = \mathbf{v}_o^2 + 2\mathbf{a}(\mathbf{x} - \mathbf{x}_o)$	missing t
Eqn. V:	$\mathbf{x} = \mathbf{x}_o + \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$	missing $\mathbf{v}_o$

The graph on the right is an acceleration versus time graph for an object moving with constant acceleration where



t_o = initial time = 0 s
$\mathbf{x}_o$ = initial position at t = 0
$\mathbf{v}_o$ = initial velocity at t = 0
t = some time later
$\mathbf{x}$ = position at some time t
$\mathbf{v}$ = velocity at some time t
$\mathbf{a}$ = acceleration of object = constant throughout motion

<u>Derivation of Equation I</u>:

Method 1) - Find the area between the acceleration vs. time graph above in terms of the variables above.

Area = ______________________________

Solving for $\mathbf{v}$: ________________________________

Eqn. I: ____________________

Method 2) - Knowing that the acceleration is constant, use the formula

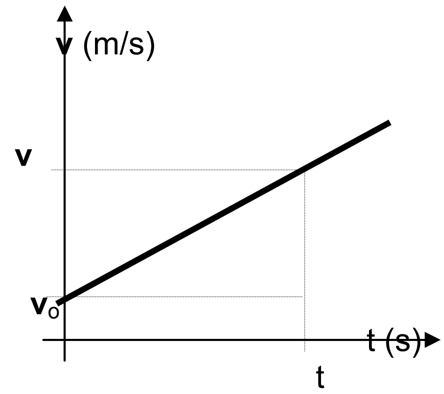
$$\Delta \mathbf{v} = (\mathbf{v} - \mathbf{v}_o) = \int_{t_1}^{t_2} \mathbf{a}(t)\, dt \qquad \text{where } \mathbf{a}(t) \text{ is constant}$$

and perform the integration to find the change in velocity in terms of the variables above. Your result should be the same as you found in Method 1).

Eqn. I: ____________________

Derivation for Equation II:

How does the graph on the right represent an object moving with constant acceleration?



____

____

Find the displacement of the object in terms of variables on the previous page between t = 0 and t = t by finding the area between the graph and the time axis. (Think trapezoid!)

Area = ____________________________________

Solving for **x**: ____________________________________

Eqn. II: ____________________

Derivation for Equation III:

Knowing that Equation I describes how the velocity of an object moving with constant acceleration varies with time, use the formula

$$\Delta \mathbf{x} = (\mathbf{x}_2 - \mathbf{x}_1) = \int_{t_1}^{t_2} \mathbf{v}(t)\, dt \qquad \text{where } t_1 = 0 \text{ and } t_2 = t$$

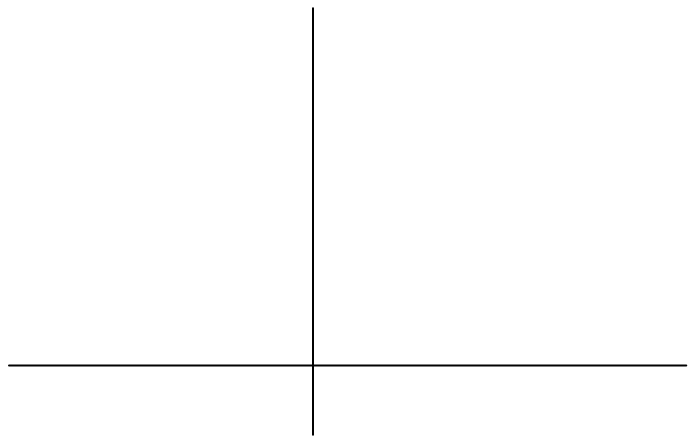
and perform the integration using the variables listed on the previous page. This result gives the displacement of an object in terms of $\mathbf{v}_o$, t, and **a**.

Eqn. III: ____________________

Equation III represents an object moving with constant acceleration. If the object starts at rest and you were to sketch a graph of position versus time for the object, what shape would the graph look like? ________________________

Using Equation III, sketch the graph of the position versus time for an object starting at rest and then moving with constant acceleration.

x	t



<u>Derivation for Equation IV</u>:
To obtain Equation IV, algebraically combine Equation I and II and eliminate t.

Eqn. IV: ____________________

<u>Derivation for Equation V</u>:
To obtain Equation V, algebraically combine Equation I and II and eliminate $\mathbf{v}_o$.

Eqn. V: ____________________

Solving Problems using Kinematics Equations

Step 1) Draw a picture of the situation and write down all given variables and quantity asked for.

Step 2) Decide on and sketch out a coordinate system.

Step 3) List the 5 kinematics variables with given values. Identify what's being asked for.

Step 4) Choose and write an equation in variable form (no numbers yet) based on which variable is NOT given in the problem.

Step 5) Do algebra for variable desired, substitute known quantities, box your answer, and write sentence describing answer.

http://www.physlink.com/fun_mcpherson.cfm

There are times when being a whiz at physics can be a definite drawback.

Using Kinematics Equations

Guido Spumone, the Infamous Italian Daredevil

Guido, the man about town, jumps into his rocket-powered sled, dons his helmet, buckles up, and ignites his engines. The sled accelerates uniformly in a straight line from rest to a velocity of 450 m/s (about 1000 mph) in 1.8 seconds.

a) How far did Guido travel in 1.8 seconds?

b) What was Guido's acceleration? [Test taking hint: Don't use answer from previous part (if possible) just in case it's wrong.]

Spanking the Lancers

During a rally before a GHS/McQHS football game, a loyal Grizzly physics student fires a tattered Lancer doll from a hovering helicopter vertically downward onto the very hard cement with her newly designed Spanking Lancer (LS) cannon. She knows that the doll leaves the LS cannon with speed 20 m/s and the helicopter is hovering 60 meters above the quad.

a) What's the velocity of the Spartan doll just before hitting the cement?

b) If a 1.5 m tall freshman sees the doll just as it leaves the cannon and is directly underneath the cannon, how much time does he have to get out of the way so that the doll doesn't hit him?

Train Collision?

Two trains heading straight for each other on the same track are 250 m apart when their engineers see each other and hit the brakes. The Express, heading west at 96 kph, slows down, decelerating at an average of 4 m/s^2, while the eastbound Flyer, traveling at 110 kph, slows down, decelerating at an average of 3 m/s^2. Will they collide?

Turtle vs. Hare

Having taken a nap under a tree only 50 m from the finish line, Rabbit wakes up to find Turtle 49 m beyond him, grinding along at 0.13 m/s. If the bewildered hare can accelerate at 9 m/s^2 up to his top speed of 18 m/s and sustain that speed, will he win?

Using Calculus

A test vehicle on a straight track begins a run starting from rest at t=0 and x=0. It subsequently has an acceleration versus time curve that starts with a linear increase and then levels of after 10 s at 5 m/s^2.

a) Determine the equation for v(t) during the time interval $0 \le t \le 10s$. Sketch v(t) for the entire journey.

Projectile Motion Summary

> Vertical motion and horizontal motion are INDEPENDENT!
> **r** changes, **v** changes, but **a** is constant with no x component

Vertical position in time

y (m)

$\mathbf{y}(t) = \mathbf{v}_o \sin\text{ø}_o t - \frac{1}{2}\mathbf{g}t^2$

Vertical velocity in time

$\mathbf{v}_y(t) = \mathbf{v}_o \sin\text{ø}_o - \mathbf{g}t$ no $\Delta\mathbf{y}$

$\mathbf{v}_y^2 = (\mathbf{v}_o \sin\text{ø}_o)^2 - 2\mathbf{g}\Delta\mathbf{y}$ no Δt

Vertical acceleration in time

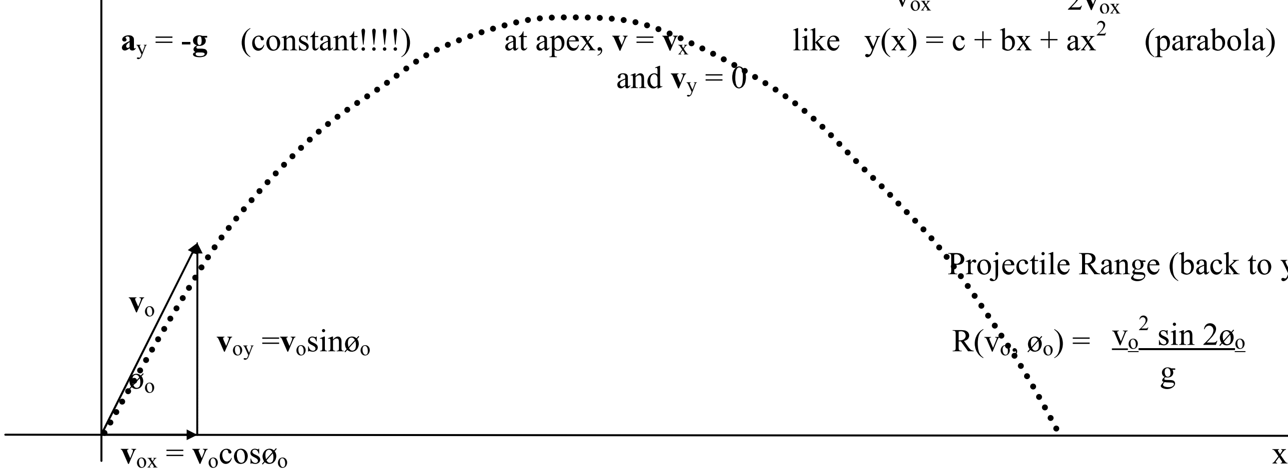
$\mathbf{a}_y = -\mathbf{g}$ (constant!!!!)

Parabolic path described as

$$\mathbf{y}(x) = \mathbf{y}_o + [\tan\text{ø}_o]x - \left[\frac{\mathbf{g}}{2\mathbf{v}_o^2\cos^2\text{ø}_o}\right]x^2 \quad \text{no } \Delta t$$

$$= \mathbf{y}_o + \left[\frac{v_{oy}}{v_{ox}}\right]x - \left[\frac{\mathbf{g}}{2\mathbf{v}_{ox}^2}\right]x^2$$

like $y(x) = c + bx + ax^2$ (parabola)



Projectile Range (back to $y = y_o$)

$$R(v_o, \text{ø}_o) = \frac{v_o^2 \sin 2\text{ø}_o}{g} \quad \text{no } \Delta t$$

Horizontal position in time $\mathbf{x}(t) = \mathbf{v}_o\cos\text{ø}_o\, t + \frac{1}{2}\overset{0}{\cancel{\mathbf{a}_x}}t^2$ $\mathbf{a}_x = 0$ and $\mathbf{x}_o = 0$

Horizontal velocity in time $\mathbf{v}_x(t) = \mathbf{v}_o\cos\text{ø}_o + \underset{0}{\cancel{\mathbf{a}_x}}t$ (constant!!!!)

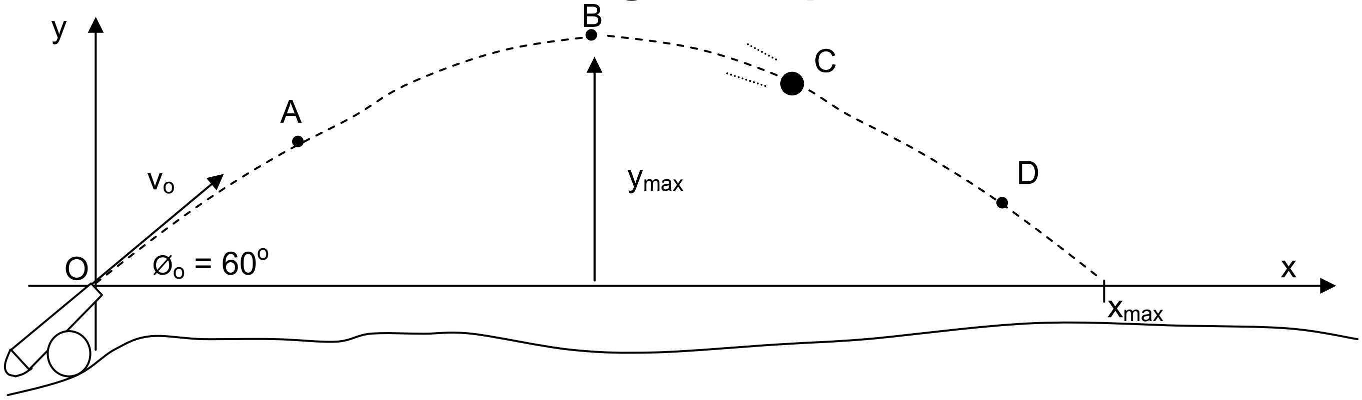
Horizontal acceleration in time $\mathbf{a}_x(t) = 0$ or $\Delta\mathbf{v}_x = 0$

> All equations assume the standard coordinate system with $g=+9.8\text{m/s}^2$

Problem solving tips

*Hang time is governed **only** by vertical motion
*Solve x and y components **independently**
*Given **initial** conditions, you can find **final** conditions
*Given **final** conditions, you can find **initial** conditions

Launching a Pumpkin



As part of Halloween festival, a large pumpkin is fired from a cannon as shown. It emerges out of the cannon at an angle of 60^o from the horizontal axis with a speed of 20 m/s. Throughout its flight, consider the pumpkin to be in freefall.

1. At which of the points O, A, B, C, or D is the magnitude of the vertical component of the pumpkin's velocity (its vertical speed) the greatest?_____the least?_______
2. At which of the points O, A, B, C, or D is the magnitude of the horizontal component of the pumpkin's velocity (its horizontal speed) the greatest?_____the least?_______
3. At which points is the magnitude acceleration the greatest?_____the least?_______
4. What is the direction of the pumpkin's acceleration at each point?__________________
5. What are v_{ox} and v_{oy}, the horizontal[4] and vertical[5] components of the pumpkin's initial velocity?

6. How high[6] does the pumpkin reach above the muzzle of the cannon?

---

[4] v_{ox}=10m/s
[5] $v_{oy}=10\sqrt{3}$m/s
[6] y_{max}=15m

7. How long[7] after being launched does the pumpkin reach the level of the muzzle of the cannon again?

8. What is the pumpkin's velocity[8] at the highest point of its trajectory?

9. What is the horizontal[9] component of the pumpkin's velocity at 3 seconds?

10. What is the vertical[10] component of its velocity at 3 seconds?

11. What is pumpkin's velocity[11] at 3 seconds?

---

[7] $t=2\sqrt{3}s=3.5s$

[8] v_{top}=10m/s, 0°

[9] v_x=10m/s

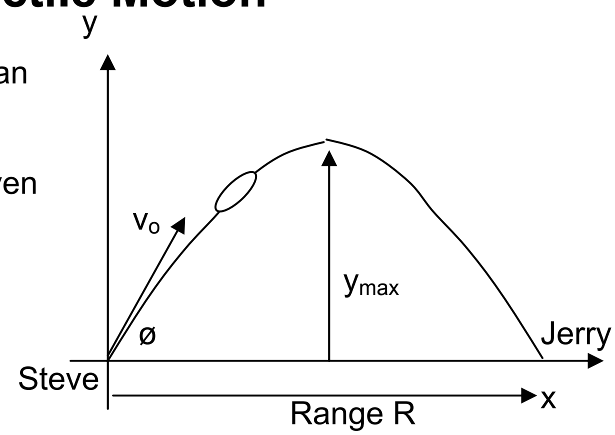
[10] v_y=-12.7m/s

[11] v=10i-12.7j m/s = 16.2 m/s, -52°

AP Physics C - Mechanics

Unit 1 KINEMATICS

49er Football Projectile Motion



Steve Young throws a football with an initial speed v_o at an angle ø relative to the horizontal ground.

1. Show that the maximum height the ball reaches is given by $y_{max} = (v_o \sin ø)^2/2g$.
   [Hint – Find either: (a) the time when $y = y_{max}$ by setting $dy/dt = 0$ then solve for t and substitute that t into y(t), or: (b) the horizontal distance x of the ball when $y = y_{max}$ by setting $dy/dx = 0$ then solve for x and substitute that x into y.]

2. Show that the time it takes for the ball to return to its launch level into Jerry Rice's hand is given by

$$t = 2v_o \sin ø \,/\, g$$

(Hint – When $t = t_{max}$, $y = 0$)

3. Show that the range, R, is given by

$$R = v_o^2 \sin 2ø \,/\, g.$$

(Hint – Find x(=R) when y = 0 and use the identity $2\sin ø \cos ø = \sin 2ø$)

4. Show that the maximum range of the ball, R_{max}, occurs when $ø = 45^o$

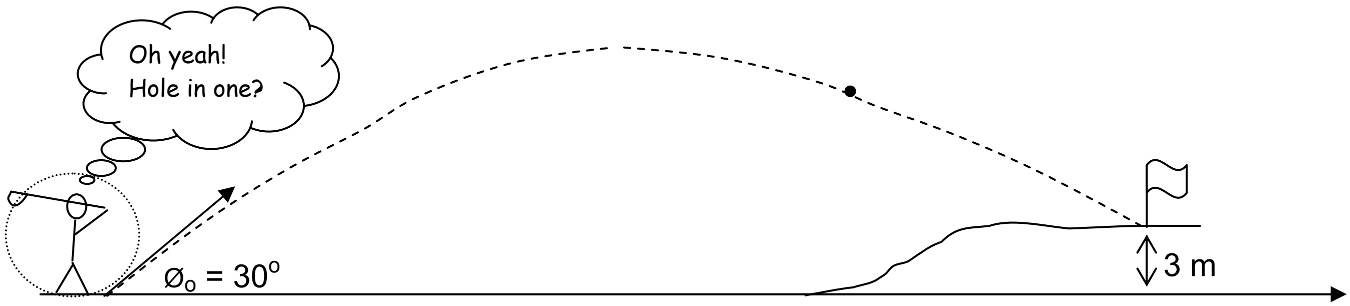
(Hint – Using #3 and the calculus max/min technique, i.e., solve $dR/dø = 0$ to find ø when $R = R_{max}$)

5. Show that two objects thrown at any two complementary angles relative to the horizontal (e.g., 70^o and 20^o or 55^o and 35^o or a line drive and a fly ball) will have the same range.

(Hint – Let $ø_1$ result in range R_1 and $ø_2$ result in range R_2. Show that if $ø_1 + ø_2 = 90^o$, then $R_1=R_2$. Recall that $\sin(180^o-ø)=\sin ø$.)

Tiger Woods Physics

Tiger drives a golf ball at an angle of 30^o with the horizontal as shown below. The green is 3 meters above the position of the ball when Tiger hits it and the hole is 160 meters away.



1. For Tiger's situation, write the following vectors in rectangular form substituting in all known quantities.

   Ball position **r**(t) = (_________________ $\hat{i}$ + _________________ $\hat{j}$) ____ unit

   Ball velocity **v**(t) = (_________________ $\hat{i}$ + _________________ $\hat{j}$) ____ unit

   Ball acceleration **a**(t) = (_________________ $\hat{i}$ + _________________ $\hat{j}$) ____ unit

2. If Tiger is to get a hole in one, what must be its minimum initial speed[12]? Write its initial velocity[13] in rectangular form.

If the ball is given the initial velocity found in #2, find the following:

3. How much time[14] after it is struck will the ball hit the green/hole?

4. When[15] will the ball reach its maximum height and what will be the maximum height[16] the ball will reach above its initial position?

---

[12] v_o=43.3 m/s
[13] $\mathbf{v}_o$=(36.8i+21.3j)m/s
[14] 4.3s
[15] 2.2s
[16] 23.1m

5. What will be the ball's velocity[17] when it reaches its maximum height?

6. How high[18] will that ball be when it is 60 meters horizontally away from Tiger?

7. How long[19] after being struck will it take to travel 60 meters horizontally away from Tiger?

8. What will be the ball's velocity[20] when it is 60 meters horizontally away from Tiger?

9. What will be the ball's velocity[21] when it first strikes the green? At what angle relative to the horizontal will it hit the green?

10. For the given initial speed, the maximum horizontal distance a projectile can reach will be when it is launched at an angle of 45^o. If Tiger smacks the ball with the initial speed found in #2 but launched it at an angle of 45^o instead of 30^o, how much farther[22] will it go horizontally before hitting the ground, assuming the ground behind the green is level with the green itself.

---

[17] **v** = 36.9i m/s
[18] 21.7m
[19] 1.6s
[20] **v** = (36.9i +5.62j) m/s
[21] **v** = (36.9i –20.8j) m/s, -29.5^o
[22] 27.9 m

Rotating Vectors in 2-D motion

Reference: Checkpoint #2, Pg. 55, HRW

The figure on the right shows a circular path taken by a particle.

1. If the instantaneous velocity of the particle is

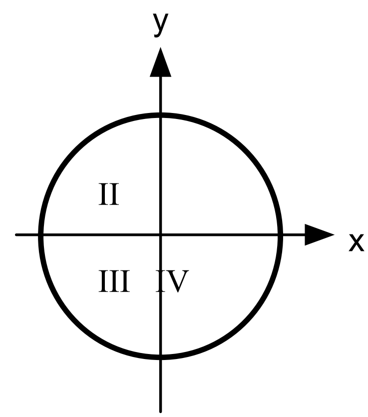
$$\mathbf{v} = (2\text{ m/s})\ \hat{i} - (2\text{ m/s})\ \hat{j}$$

through which quadrant is the particle moving when it is traveling

(a) clockwise around the circle?___________________

(b) counterclockwise around the circle?___________________

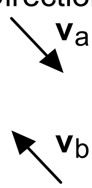
(Hint: See below before answering.)

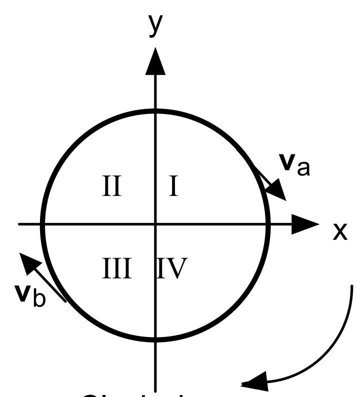


Solution: Consider general cases of various velocity vectors that are ALWAYS TANGENT to the path of motion.

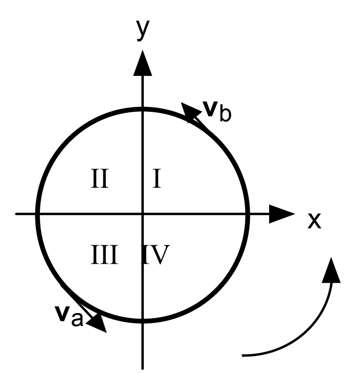
Draw vectors into the diagrams.

Vector	Direction
$\mathbf{v}_a = v_x\ \hat{i} - v_y\ \hat{j}$	
$\mathbf{v}_b = -v_x\ \hat{i} + v_y\ \hat{j}$	
$\mathbf{v}_c = -v_x\ \hat{i} - v_y\ \hat{j}$	
$\mathbf{v}_d = v_x\ \hat{i} + v_y\ \hat{j}$	
$\mathbf{v}_e = v_x\ \hat{i}$	
$\mathbf{v}_f = -v_x\ \hat{i}$	
$\mathbf{v}_g = v_y\ \hat{j}$	
$\mathbf{v}_h = -v_y\ \hat{j}$	





Clockwise



Counterclockwise

Note: This same concept applies to tangential acceleration vectors that are ALWAYS tangent to velocity versus time graphs.

2. At a particular instant of time, an object has the following vectors:

Position **r**(t) = (-2 $\hat{i}$ + 4 $\hat{j}$) m Velocity **v** = (2 $\hat{i}$) m/s Acceleration **a** = (6 $\hat{j}$) m/s^2

a) How is the magnitude and direction of **r** changing at this instant?

(decreasing, rotating CW)

b) How is the magnitude and direction of **v** changing at this instant?

(magnitude constant, rotating CCW)

A while later **r** = (4 $\hat{j}$) m, **v** = (2 $\hat{i}$ -3 $\hat{j}$) m/s, and **a** = (-4 $\hat{i}$ + 6 $\hat{j}$) m/s^2.

c) How is the magnitude and direction of **r** changing at this instant?

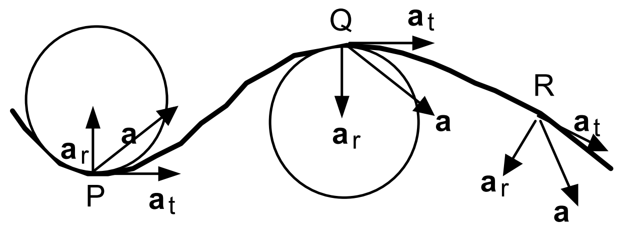
(decreasing, rotating CW)

d) How is the magnitude and direction of **v** changing at this instant?

(decreasing, not rotating)

Uniform Circular Motion

Consider the motion of a particle along a curved path where the velocity changes both direction and in magnitude as shown below. The total acceleration vector, **a**, changes from point to point as the particle moves along. This vector can be resolved into its components: a radial component vector, $\mathbf{a}_r$, and a tangential component, $\mathbf{a}_t$ where

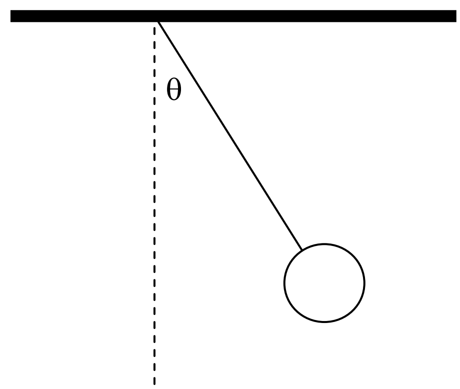


$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t \qquad |a| = \sqrt{(a_r^2 + a_t^2)}$

The <u>tangential</u> acceleration arises from the <u>change in the speed</u> of the particle. $a_t = d|v| / dt$

The <u>radial</u> acceleration is due to the <u>change in direction</u> of the velocity vector. $a_r = v^2 / r$
Also called centripetal (center seeking) acceleration.

Example: The Swinging Ball



A ball tied to the end of a string 0.5 m in length swings in a vertical circle under the influence of gravity, as shown on the right. When the ball makes an angle of $\theta = 20^o$ with the vertical, the ball has a speed of 1.5 m/s.

(a) Find the magnitude of the radial component of acceleration of this instant.

a_r = ____ / ____ = __________ / __________ = _______
(variables) (numbers)

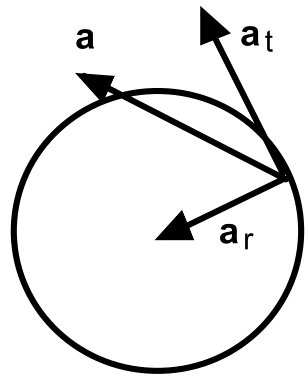
(b) When the ball is at an angle θ to the vertical, it has a tangential acceleration of magnitude gsin θ (produced by the tangential component of the force mg.) When the ball makes an angle of $\theta=20^o$, $a_t = g \sin 20^o = 3.4 m/s^2$, find the magnitude and direction of the total acceleration.

Solution: The magnitude can be found with $|a| = \sqrt{(\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_)}$ (variables)

$= \sqrt{(\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_)}$ (numbers)

= _____________

The direction can be found with $\theta = \tan^{-1}$ (____/ ____) = $\tan^{-1}$ (_______/ ________) = __________
(variables) (numbers)



For the special case of Uniform Circular Motion (UCM) as shown on the right, the term “uniform” implies that the object is moving with constant _____________, and therefore the ________________acceleration is zero.

Thus it’s only acceleration is directed _____________________ otherwise known as __________________________________ and has magnitude of _______.

The **period** (time) of travel around the circumference exactly once is given by $\boxed{T = 2\pi r / v}$

Angular speed is defined as the change of angle (in radians) per unit time $\boxed{\omega = \Delta\theta / \Delta t}$ [**rads**/sec] or rather answers the question “how fast is the object rotating?”.

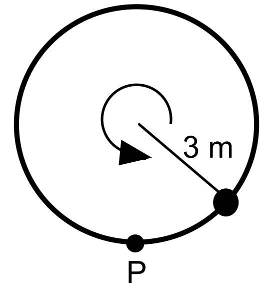
Linear speed (magnitude of the linear velocity) is defined as $\boxed{v = \omega r}$ where points with greater radius on a rotating object have greater linear speed.

PRACTICE PROBLEMS

1. The moon’s nearly circular orbit about the Earth has a radius of about 384,000 km and a period of 27.3 days. Determine the acceleration[23] of the moon toward the Earth.

[23] ($a_r = 2.72 \times 10^{-3}$ m/s^2)

2. An object moves in a circle of radius 3 m at a constant speed, completing 3 revolutions every minute.
At t = 0, it is at point P and is moving counter-clockwise.



a) Find its period[24] T in seconds.

b) Find its angular[25] speed in rad/s.

c) Find its linear[26] speed.

d) Find the magnitude of its acceleration[27].

[24] 20s
[25] $\omega = \pi/10$ rad/s
[26] $v = 0.3\pi$ m/s
[27] $a = 0.03\pi^2$ m/s^2

Two Dimensional Motion

Supplementary Problems

1. An object has a time dependent position vector given by $\mathbf{r}(t) = (0.5\,t^3\,\hat{i} + t^2\,\hat{j})$ m.

a) Draw a graph of the trajectory of the object for $0 \le t \le 3$ seconds and indicate on the graph the direction of motion of the object.

b) Determine the speed of the object at t = 1 s.

(2.5 m/s)

c) Determine the magnitude of the acceleration of the object at t = 2 s.

(6.3 m/s^2)

2. Given the following x and y components of the position of an object as a function of time

$x = (3\,t - 4\,t^2)$ $y = (-6\,t + t^3)$ where x and y are in meters and t in seconds.

a) Find the object's velocity as a function of time.

{ $\mathbf{v}(t) = [(3-8t)\,\hat{\imath} + (-6 + 3t^2)\,\hat{\jmath}]$ m/s}

b) Find the object's acceleration as a function of time.

{ $\mathbf{a}(t) = [(-8)\,\hat{\imath} + (6t)\,\hat{\jmath}]$ m/s^2}

c) Find how far the object is from the origin at t = 3 s.

(28.5 m)

d) Find the object's speed at t = 0.

(6.7 m/s)

e) Find the magnitude of the object's acceleration at t = 3 s.

(19.7 m/s^2)

f) Draw the position, velocity, and acceleration vectors at t = 2 s.

3) Given that the coordinates of an object are (1.0 m, 2.0 m) at t = 0 and the x and y components of its velocity at any time are given by $v_x = 4t^3 + 4t$ and $v_y = 4t$, where v is in m/s and t in seconds,

a) Show that the equation of the trajectory of the object is given by $x = y^2/4$.

b) Draw the position, velocity, and acceleration vectors for the object at t = 1 s.

4) Given that the position of an object at t = 0 is $\mathbf{r}_o = (-3\ \hat{j})$ m, its velocity at t = 0 is $\mathbf{v}_o = (4\ \hat{i})$ m/s, and the x and y components of its acceleration as a function of time are given by $a_x = -8 \sin 2t$ and $a_y = 12 \cos 2t$, where t is in seconds.

a) Find the object's acceleration at $\pi/8$ seconds.

$[\mathbf{a}_{\pi/8} = (-4\sqrt{2}\ \hat{i} + 6\sqrt{2}\ \hat{j})\ \text{m/s}^2]$

b) Find the magnitude of the object's acceleration at $\pi/4$ seconds.

$(\mathbf{a}_{\pi/4} = 8\ \text{m/s}^2)$

c) Find the object's velocity as a function of time.

$\{\mathbf{v}(t) = [(4 \cos 2t)\ \hat{i} + (6 \sin 2t)\ \hat{j}]\ \text{m/s}\}$

d) Find the object's velocity at $\pi/4$ seconds.

$(\mathbf{v}_{\pi/4} = 6\ \hat{j}\ \text{m/s})$

e) Find the object's speed at $5\pi/8$ seconds.

($\mathbf{v}_{5\pi/8} = \sqrt{26}$ m/s)

f) Find the object's position as a function of time.

{$\mathbf{r}(t) = [(2 \sin 2t)\ \hat{i} - (3 \cos 2t)\ \hat{j})]$ m}

g) Find the object's position at $\pi/8$ seconds.

{$\mathbf{r}_{\pi/8} = [(\sqrt{2}\ \hat{i}) - (3\sqrt{2}/2\ \hat{j})]$ m }

h) Find how far the object is from the origin at $\pi/3$ seconds.

($\mathbf{r}_{\pi/3} = \sqrt{21} / 2$ m)

i) Find the equation of the trajectory of the object.

($x^2 / 4 + y^2 / 9 = 1$)