Name:________________________________________________Per:__________

AP Physics C

Semester 1 - Mechanics

# Unit 3
# Forces with Friction & Circular Motion
# Workbook

# Unit 3 - Forces with Friction & Circular Motion

Supplements to Text Readings from
Fundamentals of Physics by Halliday, Resnick, & Walker **Chapter 6**



# Unit 3 - Objectives and Assignments

TEXT: *Fundamentals of Physics* by Halliday, Resnick, & Walker, **Chapter 6**

**All objectives from Unit 2 apply in this unit with the addition of the following:**

**II. Dynamics of a Single Body with friction (Second Law)**

a. Students should understand the significance of the coefficient of friction so they can:

(1) Write down the relationship between the normal and frictional forces on a surface.

(2) Analyze situations in which a body slides down a rough inclined plane or is pulled or pushed across a rough surface.

(3) Analyze static situations involving friction to determine under what circumstances a body will start to slip, or to calculate the magnitude of the force of static friction.

b. Students should understand the effect of fluid friction on the motion of a body so they can:

(1) Find the terminal velocity of a body moving vertically through a fluid that exerts a retarding force proportional to velocity.

(2) Describe qualitatively, with the aid of graphs, the acceleration, velocity, and displacement of such a particle when it is released from rest or is projected vertically with specified initial velocity.

c. Students should be able to analyze situations in which a body moves under the influence of one or more forces with a specific acceleration so that they can determine the magnitude and direction of the net force, or of one of the forces that makes up the net force, in situations such as the following:

(1) Motion in a horizontal circle (e.g., mass on a rotating merry-go-round, or car rounding a banked curve).

(2) Motion in a vertical circle (e.g., mass swinging on the end of a string, cart rolling down a curved track, rider on a Ferris wheel

**III. Systems of Two or More Bodies (Third Law)**

a. Students should be able to solve problems in which application of Newton's Laws leads to two or three simultaneous linear equations involving unknown forces or accelerations.

Mechanics Unit 3 Homework

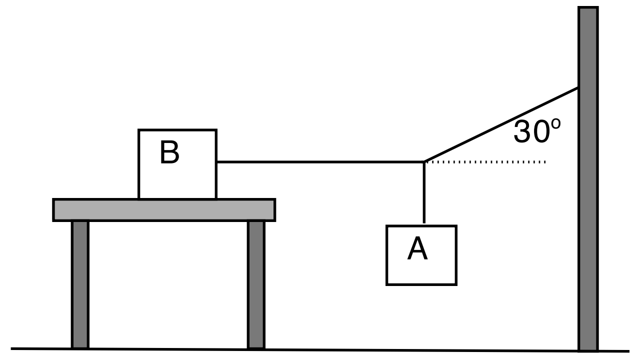
Chapter 6 #1, 2, 3, 4, 6, 9, 11, 14, 15, 17, 18, 19, 21, 22, 23, 25, 26, 33, 25, 27, 28, 29, 30, 32, 34, 35, 37, 38, 44, 45, 46, 48, 52, 55, 57, 61, 63

# Newton's Laws with Friction

Supplementary Problems

## 1. Blocks and Strings

From Pg. 126 #31 - Block B weighs 711 N. The coefficient of static friction between the block and horizontal surface is 0.25.




(a) Draw the free body diagrams for each mass and the cord junction.

(b) Write Newton's Laws equations.

(c) Find the maximum weight[1] of block A for which the system will be stationary.

(d) Given that $m_A$ = 50 kg, now find the mass[2] of block B that will keep the system stationary.

[1] 103 N
[2] 346 kg

## 2. Puck on Ice

A hockey puck on a frozen pond is hit and given an initial speed of 20 m/s. The puck always remains on the ice and slides 115 m before coming to rest

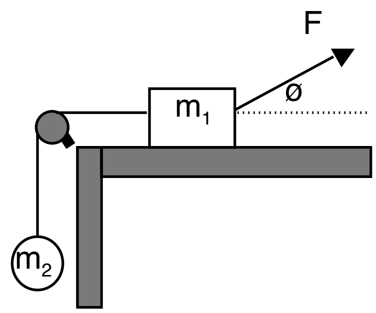
a) Draw the diagram and the free body diagrams for the puck. (b) Write Newton's Laws equations.

(c) Determine the coefficient of kinetic friction[3] between the puck and ice.

[3] 0.177

## 3. Blocks and Pulleys

A mass $m_1$ = 15 kg on a rough, horizontal surface is connected to a second mass $m_2$ = 10 kg by a lightweight cord over a lightweight, frictionless pulley as shown on the right. A force of magnitude F = 250 N at an angle ø = 25° with the horizontal is applied to $m_1$ and the coefficient of kinetic friction between $m_1$ and the surface is $\mu_k$ = 0.3.




a) Draw the free body diagrams for each mass.

(b) Write Newton's Laws equations

(c) Determine the magnitude of the acceleration[4] of the masses.

(d) Determine the tension[5] in the cord.

[4] 4.6 m/s²
[5] 144 N

## 4. Skier on Hill

A skier has just begun descending down a $30^o$ slope. Assume that the coefficient of kinetic friction between the skis and the snow is 0.10.

(a) Draw a diagram and the free body diagram for the skier.

(b) Write Newton's laws equations.

(c) Calculate her acceleration[6]. What can you conclude about the dependence of mass on acceleration?

(d) Calculate the speed[7] she will reach after 4.0 s.

(e) Suppose, instead, the snow is slushy and she moves down the $30^o$ slope at constant speed. What can you say about the coefficient of friction, $\mu_k$?

[6] 4 $m/s^2$
[7] 16 m/s

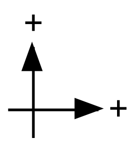
## 5. Blocks in Motion

Cases I through IV in the table below represent 4 states of motion for a block on a horizontal surface. Using the blocks below the table, **draw the free-body diagrams** showing all the forces acting on the block in each case. Using the quantities given, fill in the blanks in the table.

**Attach all your calculation work for each case**.

| Quantity | Case I | Case II | Case III | Case IV |
|---|---|---|---|---|
| Velocity v | 0 | constant | changing | changing |
| Mass M | 2 kg | | | 9 kg |
| Weight W | | 35 N | 60 N | |
| Normal Force N | | | 60 N | |
| Kinetic Friction $f$ | 0 | | 24 N, left | |
| Coefficient of kinetic friction $\mu_k$ | N/A | 0.2 | | 0.3 |
| Applied Force $F_a$ | 5 N, down | | | 40 N right |
| Net Force $F_n$ | | | | |
| Acceleration a | | | 0.33 m/s$^2$, right | |

Coordinate system:




**Case I**

**Case II**

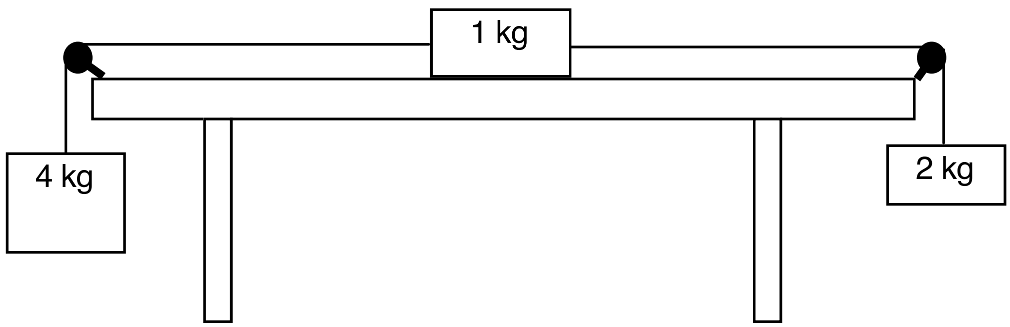
**Case III**

**Case IV**

## 6. Solving problems with simultaneous equations

Three masses are connected on the table as shown below. The table has a coefficient of sliding friction of 0.35.

a) Determine the acceleration[8] of each block and their directions.
b) Determine the tensions[9] in the two cords.




[8] 2.3 m/s², right

[9] 30 N, 24N

## 7. Drag Force / Terminal Velocity - Skydiver

A skydiver of mass 80 kg jumps from a slow moving aircraft and reaches her terminal velocity of 50 m/s.

a) When she reaches a velocity of 30 m/s, what is the general expression for her acceleration and what is the magnitude of her acceleration[10]?

b) What is the drag force[11] on the diver when her speed is 50 m/s?

c) What is the drag force[12] on the diver when her speed is 30 m/s?

---

[10] 6.3 $m/s^2$
[11] 784 N
[12] 282 N

## 8. Drag Force / Terminal Velocity – Dropping a Ball

a) Estimate the terminal velocity[13] of a wooden sphere with density d = 0.83 g/cm$^3$ moving in air if its radius is 8 cm. (Density of air ≤ 1.21 kg/m$^3$)

b) From what height[14] would a free-falling object reach this speed with no air resistance?

[13] 53.6 m/s
[14] 146 m

# Friction Force Proportional to Velocity

A ping pong ball of mass m is released from rest. As it falls, air friction exerts a retarding force on the ball that is proportional to its speed ( i.e. $F_r = kv$ where k is some constant). As the ball falls, its speed increases and therefore, so does the retarding frictional force eventually causing its acceleration to become zero. After is ceases to accelerate, the ball falls with constant terminal velocity $v_t$.

a. In the spaces provided below, draw FBDs showing all the force on the ball

| i) Just after it is dropped | ii) After is has been falling for a long time and has reached $v_t$ |
|---|---|

b. Write Newton's 2nd Law for case i) above and show that the acceleration of the ball just after it is dropped is "g".

c. Write Newton's 2nd Law for case ii) above and show that after the ball has been falling for a long time, its terminal velocity becomes $v_t = mg/k$

d. Choosing down as positive, show that the velocity of the ball as a function of time is

$$v = \frac{mg}{k}\left(1 - e^{-\frac{k}{m}t}\right).$$ (Hint: Use "Separation of Variables")

i) Write Newton's 2nd Law for the motion of the ball while it falls, solve your expression for the acceleration, and then replace the acceleration with dv/dt.

ii) "Separate the variables" v and t by putting everything involving v and all constants on the left hand side of the equal sign and dt on the right hand side.

iii) Integrate both sides of your above equation. The right hand side should be integrated from 0 to t and the left hand side from 0 (since the ball started from rest) to v (the velocity at any time t).

iv) Use your knowledge of the properties of logarithms and solve v as a function of time.

v) Use the expression found in iv) to show that after a long time (t □ • ) velocity v becomes $v_t$ as found in part c.

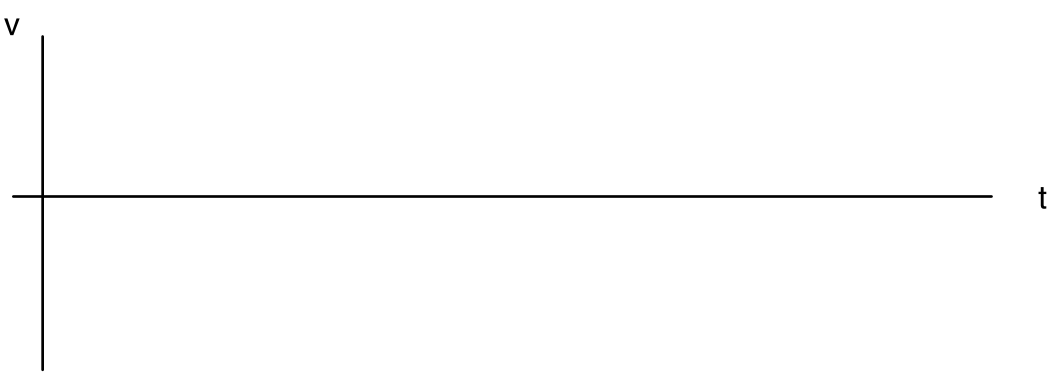
e) Show that the expression for the acceleration of the ball as a function of time is

$$a = g(e^{-\frac{k}{m}t})$$

f) Use the expression found above to show that the acceleration of the ball just after it is dropped (at $t = 0$) is the same as that found in part b.

g) Use the expression found in part e to determine the acceleration of the ball after a long time ($t \to \infty$)

h) Use all of the above to help you sketch the velocity and acceleration of the ball as a function of time.

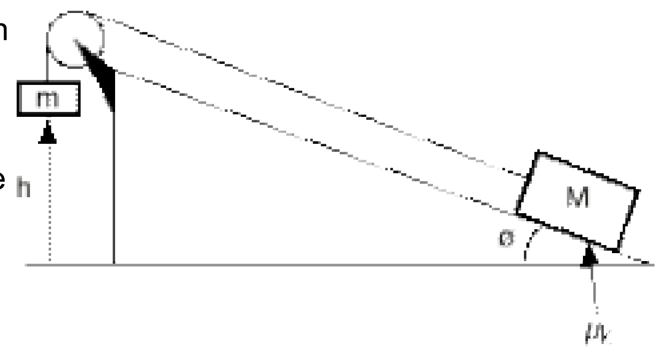



# Inclined Plane Lab

**Purpose:**

-To determine the coefficient of kinetic friction, $\mu_k$, between an object and a surface.
-To determine the tension in the string between two objects.

**Pre-lab Theory:**




1. Using kinematics, determine the equation in variable form for the acceleration of the system.

2. Draw the free body diagrams showing the forces on both m and M. Indicate the coordinate system to be used for each diagram.

3. Write Newton's 2nd Law equations in variable form along both coordinate axes for each mass.

4. Solve your equations in #3 for the coefficient of kinetic friction $\mu_k$ in variable form.

5. Assuming you have solved for $\mu_k$, solve one your equations in #3 for the tension in the string between the two masses while in motion. Have your instructor check your equations for $\mu_k$ and tension before setting up your equipment.

**Procedure & Data**

Your equations from your pre-lab theory will determine how to gather data and how your data will appear.

**Analysis:**

1. In order for your calculations to be valid, what are you assuming about the acceleration of the two masses? How did you determine and control this assumption in your experiment?

2. What is the coefficient of kinetic friction, $\mu_k$, between the object M and the surface of the inclined plane? What is the tension in the string between the two masses M and m? Do your calculations seem valid?

3. How did you determine the angle of incline for your surface?

4. For this same apparatus, theoretically determine the value of m that will cause the box M to move up or down the inclined plane at constant speed.

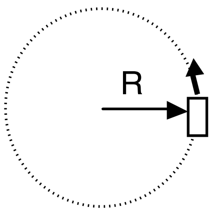
5. Discuss systematic errors, not mistakes.

# Circular Motion

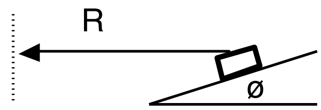
## 1. Identifying Centripetal Forces

Draw the free body diagram for each of the following and identify the centripetal force.

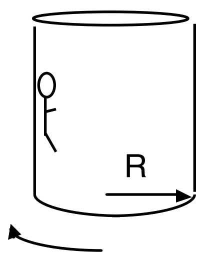
A) A car moving in a circle at constant speed.



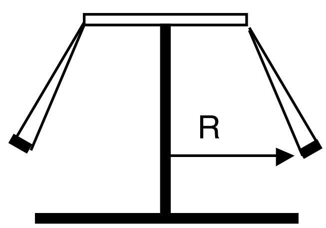

B) A car moving at constant speed around a circular BANKED track.



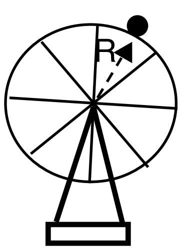

C) A student "stuck" to the wall on the Spin Out at Great America.



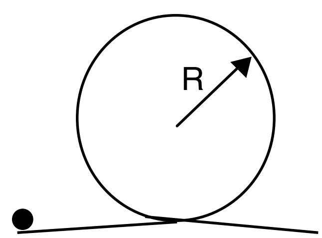

D) Students riding the rotating swings at Great America.



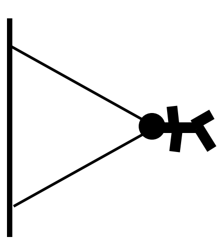

E) A student on a Ferris wheel.



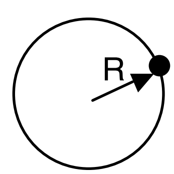

F) A daredevil attempting a death-defying vertical loop on a motorcycle.




G) A teddy bear being rotated at constant speed in a horizontal circle.



H) A rubber ducky moving in a vertical circle with non-uniform circular motion.



## 2. Unbanked Curves

From Pg. 128 #51 - A car weighing 10.7 kN and traveling at 13.4 m/s attempts to round an unbanked curve with a radius of 61 m.

(a) Draw the diagram and the free body diagrams for the car.

(b) Write Newton's Laws equations

(c) What force of friction[15] is required to keep the car on its circular path?

(d) If the coefficient of static friction between the tires and the road is 0.35, is the attempt at taking the curve successful? Show why.

[15] 3214 N

## 3. Banked Curves

This is a variation on Sample Problem 6-11 on pg. 119-120 and #53 on pg. 128 in your text.

A circular highway curve of radius 200 meters is banked at an angle such that a vehicle traveling 60 km/h can negotiate the curve in the absence of friction.

(a) Draw the diagram and the free body diagram for the car.

(b) Write Newton's Laws equations

(c) What is the angle[16] between the highway surface and the horizontal?

(d) Suppose a young Silicon Valley Dot Commer is driving his flashy BMW around the curve at a speed of 40 km/h one rainy day. Draw a free body diagram showing all the forces on the BMW as it rounds the curve and in terms of the forces in the diagram (no numbers) state the centripetal force acting on the BMW.

[16] about 8°

(e) Determine the minimum value of the coefficient of friction between the tires and the pavement necessary to prevent the BMW from skidding.

## 4. More Banked Curves

Suppose a curve with radius of 70 m is perfectly banked for a car traveling 60 km/h.

(a) Draw the diagram and the free body diagram for the car.

(b) Write Newton's Laws equations.

(c) What must be the coefficient of static friction[17] for a car not to skid when traveling at 90 km/h?

[17] 0.37

## 5. Even More Banked Curves

A 1200 kg car rounds a curve of radius 70 m banked at an angle of $12^{o}$. If the car is traveling at 90 km/h, will a friction force[18] be required? If so, how much and in what direction?

[18] 8035 N

## 6. Yet Another Banked Curve

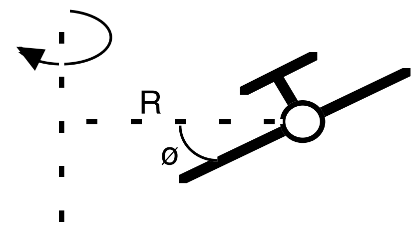
A curve of radius 60 m is banked for a design speed of 100 km/h. If the coefficient of static friction is 0.30 (wet pavement), at what range of speeds[19] can a car safely make the curve?

[19] 140 km/h ≥ v ≥ 74 km/h

## 7. Flying Loops

A vomiting MTV Road Rules Selectee and a 70 kg Professional Pilot are in an airplane flying at a constant speed of 400 km/h in a horizontal circle of radius R = 2 km. Since the aerodynamic lift force is perpendicular to the wings, the pilot must bank the plane to provide the centripetal force necessary to perform the maneuver.

a) At what angle[20] relative to the horizontal must the wings be banked?




b) What is the pilot's apparent weight[21]?

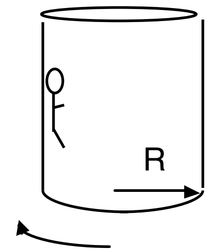
c) Now suppose the pilot wants to really freak out the weak stomached college student and changes course to fly the plane in a vertical loop of radius 1 km. If the apparent weight of the pilot at the top of the loop is half his actual weight, what is his speed[22] at the top?

d) What is the pilot's apparent weight[23] at the bottom of the vertical circular loop, if the speed of the plane is the same at the bottom as at the top?

[20] 32°
[21] 811 N
[22] 436 km/h = 121 m/s
[23] 1711 N

## 8. The Rotor

The Rotor, an amusement park ride, shown on the right consists of a rotating vertical cylinder. A rider enters, stands against the wall, and when the cylinder is rotating sufficiently fast, the floor drops down, leaving the rider "stuck" to the wall. The rider shown has mass 50 kg and the radius R of the cylinder is 5 meters.




a) Draw a free body diagram showing all the forces on the rider.

b) List all the forces in the free-body diagram and state the reaction force for each.

<u>Action</u> <u>Reaction</u>

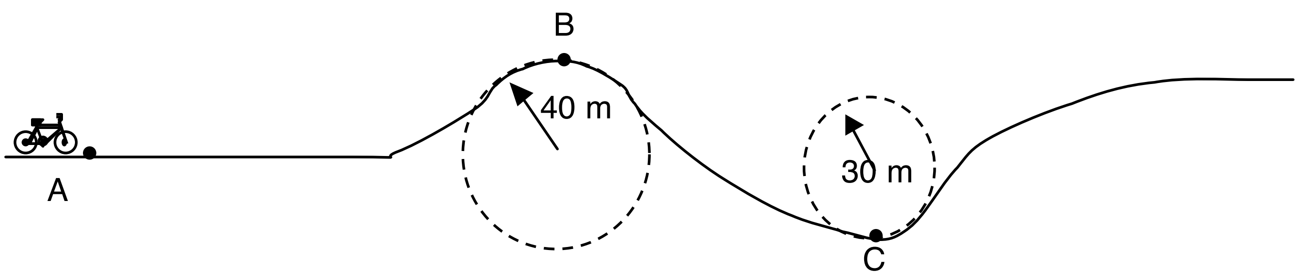
c) When the angular speed $\omega$ of the cylinder is 2 rads/s, identify what force(s) is/are providing the centripetal acceleration[25] and determine its magnitude.

d) Determine the minimum value of the coefficient of static friction[24], $\mu_k$, between the rider and the cylinder to prevent the rider from sliding down the wall when $\omega$ = 2 rad/s.

[24] 0.49
[25] 1000 N

## 9. Riding Hills and Valleys

As an extra credit project, a student in desperate need of points agrees to ride his bike at a constant speed of 15 m/s over a hilly road while sitting on a bathroom scale. The profile of the road is shown below. As he passes point A, he looks down to see that the scale reads 500 N.




a) What will the scale[26] read at points B and C?

b) What would his speed[27] have to be at point B if the scale were to read zero?

c) What would happen if the bike rider were to travel the road at 25 m/s?

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[26] 213 N, 882 N
[27] 19.8 m/s

## 10. Ferris Wheels

Similar to Pg. 129, #62 in text. A 50 kg physics student rides a Ferris wheel while sitting on a bathroom scale. The Ferris wheel rotates with a consistent angular speed, $\omega$. As she reaches maximum height, she notices the bathroom scale reads 420 N.

a) What is the magnitude[28] and direction[29] of her centripetal acceleration at her highest point?

b) What does the scale[30] read at her lowest point?

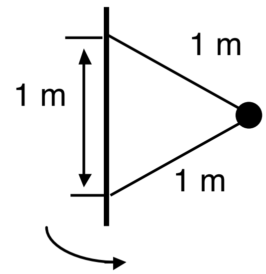
c) If $\omega$ were doubled, what would the scale[31] read when she is at the top?

---

[28] 1.4 $m/s^2$
[29] down
[30] 560 N
[31] 210 N

## 11. Ball & Strings

A 1 kg ball is attached to a vertical rod by two strings each 1 m long. The strings are attached to a rod at points 1 m apart. The rod-strings-ball system is rotating about the axis of the rod with both strings taut and forming an equilateral triangle with the rod as shown below. The tension in the upper string is 25 N.




a) Draw a free body diagram showing all the forces on the ball.

b) List the Action-Reaction forces on your free body diagram.

Action | Reaction

c) What is the tension[32] in the lower string?

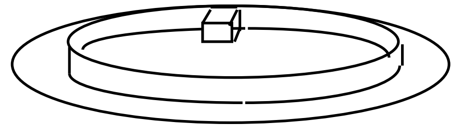
d) What is the net force[33] on the ball when the ball is in the position shown above?

e) What is the speed[34] of the ball?

[32] 5.4 N
[33] 26.9 N
[34] 4.8 m/s

## 12. Sliding Around

A small block of mass m slides on a horizontal frictionless surface as it travels around the inside of a cylinder of radius R. The coefficient of friction between the block and the cylinder is $\mu_k$. Therefore, the speed v of the block decreases. In terms of m. R. $\mu_k$, and v, find the expression for each of the following:



a) What is the friction force[35] $f$ on the block?

b) Calculate the tangential acceleration[36] of the block.

c) Determine the speed[37] v of the block as a function of time. Let $v_o$ be the speed of the block at t = 0.

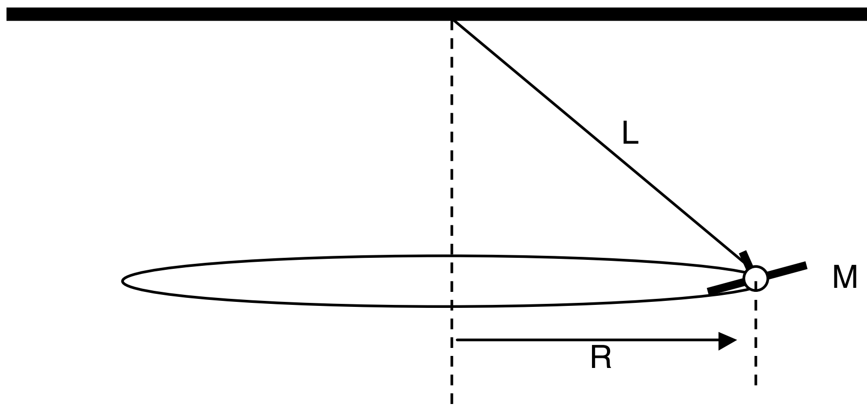
d) How much time[38] is required to reduce the speed of the block from $v_0$ to $v_o/3$.

[35] $\mu_k mv^2/R$
[36] $\mu_k v^2 / R$
[37] $Rv_o /(R - \mu_k t v_o)$
[38] $-2R/\mu_k v_o$

# Conical Pendulum Lab

**Purpose:**

- To compare the theoretical and experimental value for the period of rotation of a toy plane traveling in a uniform circular path.
- To determine the tension in the string attached to the toy plane as it travels in its uniform circular path.
- To determine the linear speed and centripetal acceleration of the toy plane.




**Procedures:** You create the procedure based on the purpose. Be sure to include you theory that governs the motion.

**Data:** Mass of toy plane M = ___________________

Length of string L = ___________________

Radius of rotation R = _____________________

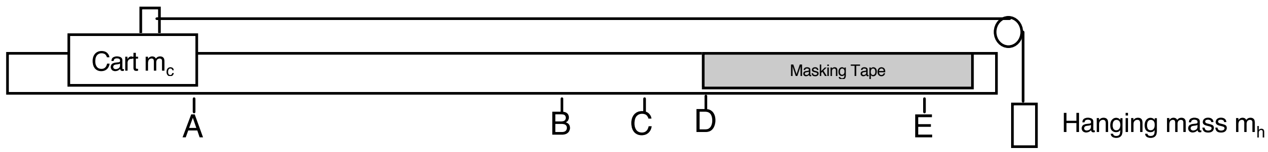
Organize other data that your group deems necessary to obtain.

**Analysis:** Show that you have an in depth understanding of the situation by effectively communicating all FBDs, equations, derivations and numerical values as required by the purpose of this lab. Discuss sources of error, not mistakes.

# Coefficient of Kinetic Friction Lab

**Purpose:** 1) To compare the theoretical acceleration of the air track cart with the calculated acceleration from experimental results.
2) To determine the coefficient of kinetic friction between the air track cart and the masking tape on the air track.

**Materials/Procedures:** Your instructor will set-up the following apparatus in class.




What happens?

- Point A - cart starts from rest, when $m_h$ is released, the cart trips the photogate at this position, starts the computer clock and the cart accelerates forward
- Point B - the cart trips the photogate at this position and stops the computer clock
- Point C - $m_h$ reaches the ground
- Point D - cart begins to slide onto the masking tape on air track
- Point E - cart comes to a stop here

**Data:** Cart mass $m_c$ = ________________ Hanging mass $m_h$ = _________________

Distance between A and B $d_{A\rightarrow B}$= __________________

Distance between B and C $d_{B\rightarrow C}$= __________________

| Trial | Time between A and B<br>$t_{A\rightarrow B}$ (s) | Distance between D and E<br>$d_{D\rightarrow E}$ (m) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| Ave. | | |

**Analysis:** (To be done in pairs)

Problem 1 - To compare the theoretical acceleration of the air track cart with the calculated acceleration from experimental results.

Your analysis should include 3 parts

- Part 1 - using only $m_c$ and $m_h$, show all FBDs, Newton's $2^{nd}$ Law equations, a derived equation for the acceleration of the cart in terms of g, $m_c$ and $m_h$, and your theoretical value for the acceleration of the cart
- Part 2 -using the measured distance $d_{A\to B}$ and measured time $t_{A\to B}$, show all derivations for the equation for the acceleration of the cart in terms of g, $d_{A\to B}$ and $t_{A\to B}$, and your experimental value for the acceleration of the cart
- Part 3 - Show how you would calculate the % difference between your theoretical and experimental values for the acceleration of the cart

Problem 2 - To determine the coefficient of kinetic friction between the air track cart and the masking tape on the air track.

You analysis should be clear, concise and present a solution to the problem worthy of AP Physics C students who are considered the best of the best at GHS.

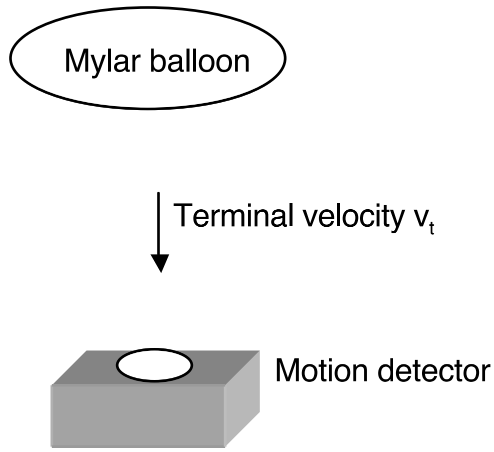
A formal lab report is expected from your pair group.

# Aerodynamic Drag Lab

**Purpose:** To determine the effect of velocity on air resistance for a falling balloon.

**Materials:** Motion sensors, computer interface, computer, mylar balloon, pennies, tape, electronic mass scale

**Procedures:** Theoretically solving the problem first will help your group determine the procedures and data for this lab. Think about what you are trying to determine BEFORE setting up equipment and taking data. Terminal velocity of a falling balloon should provide linear graphs to analyze.




**Analysis:**

Think about how best to present your results and the meaning of the data. This lab lends itself quite well to graphical interpretations of data. For any calculated regressions, be sure to include coefficients of correlation when possible.